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A Transformer Analogy for Understanding Eddy Current Effects in Loudspeaker Voice Coils

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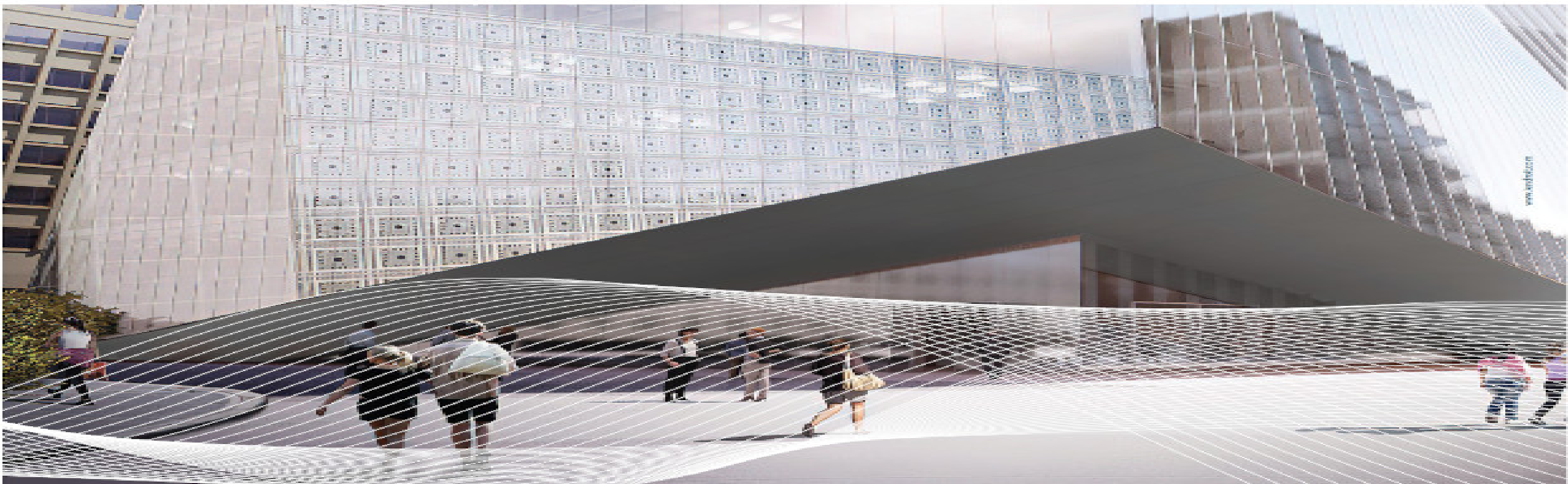


Understanding eddy currents in loudspeaker voice coils is often difficult due to the complexity of conventional semi-inductance models derived from Maxwell's equations. These models provide an accurate but mathematically intensive approach, making them less accessible to students and newcomers in the field. In this paper, the phenomenon of eddy currents is simplified using an analogy with a transformer, where the voice coil is the primary winding and the surrounding conductive materials form a short-circuited secondary winding. Using this approach, we clarify the behaviour of impedance as a function of frequency. By building on familiar transformer principles, this method makes the underlying physics more accessible, helping students to understand the complex phenomena related to eddy currents and improving the teaching of advanced loudspeaker modelling.

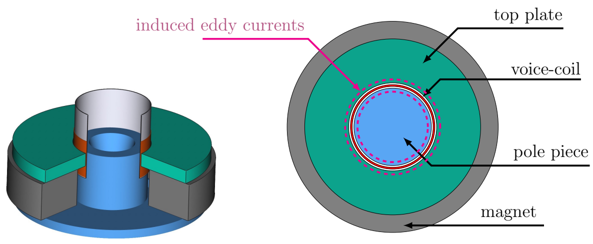
1 Introduction

Many students, new researchers, and engineers in the field of electroacoustics and audio find it often difficult to understand how eddy currents affect the loudspeaker behaviour. On the one hand, it is widely known that eddy currents are electric currents which flow in the conductive parts of the loudspeaker motors, e.g. the pole pieces (see Fig. 1). They are induced because of a change in the magnetic flux created by the voice-coil. These eddy currents then produce their own varying magnetic flux that interacts with the flux produced by the voice-coil. On the other hand, the effect of the eddy currents on the measured quantities, for instance, on the blocked impedance measurements, is not always intuitive. The mathematical formulation of this phenomenon, derived from Maxwell's equations, is complex, and the link between the mathematical descriptions and the observed measured impedance is easy to lose.

There exist many models that can be used to simulate the eddy currents influence on voice-coil behavior, and to predict the impedance measurements results. Many of them are based on a semi-inductance approach [1, 2, 3, 4] derived directly from Maxwell's equations. Other works [5, 6] propose models that use circuit analogies. While accurate in predicting the voice-coil behavior in presence of eddy currents, these models do not help to explain the observed behavior. An alternative approach uses transformer analogies [5, 7, 8]. It brings a more intuitive approach to describe the effect of eddy currents by modeling the voice-coil impedance using a transformer equivalent, representing the voice-coil as the primary winding and the conductive parts of the pole pieces as a one-turn secondary winding.

The transformer analogy is helpful in understanding the concept of eddy currents for two reasons. First, the theory of ideal transformers is usually well known by electroacoustic students and engineers. Second, the theory can be easily verified using a simple experiment with two coils or with a coil and surrounding conductive material [8]. While students find this analogy helpful, some still have difficulty understanding how non-ideal factors affect the blocked impedance voice-coil response.

In this paper, we focus on the influence of eddy currents on blocked impedance measurements of loudspeaker voice-coils. We use the transformer analogy that helps to better understand the observed impedance measurement thanks to similarities between the behavior of real-world transformers and the eddy currents in loudspeakers. The goal of the paper is to make the understanding of both the physics and the observed blocked impedance measurements easier.



(a) isometric view (b) top view

Figure 1: Schematic representation of eddy currents in an electrodynamic loudspeaker.

The paper is structured as follows. In Section 2, the influence of eddy currents on the apparent resistance and inductance is discussed, and the transformer analogy is introduced using no mathematical derivation. In Section 3, the transformer analogy is presented using a mathematical derivation of the input impedance of a transformer, and a simulation of the blocked impedance is presented. Finally, in Section 4, we discuss the limitations of the transformer analogy.

2 Eddy Currents

Eddy currents are electric currents, i.e. the flow of free electrons in a material, that are induced within a conductor by a changing magnetic field. These eddy currents generate their own magnetic fields in opposition to the original field. This phenomenon is known as Lenz's Law. It states that the direction of an induced current is such that it opposes the change that produced it. This effect can be more noticeable at higher frequencies, where the changes of magnetic field are faster.

The presence of the magnet in a loudspeaker motor has almost no effect on eddy currents. The permanent magnet creates a static magnetic field that is important for the loudspeaker operation, but as the field does not change, no eddy currents are created. The voice-coil, on the other hand, creates a changing magnetic field because of the alternative current flowing through it, and this changing field induces eddy currents in the conductive parts of the motor. The presence of eddy currents can be demonstrated using a simple measurement of the blocked impedance. In

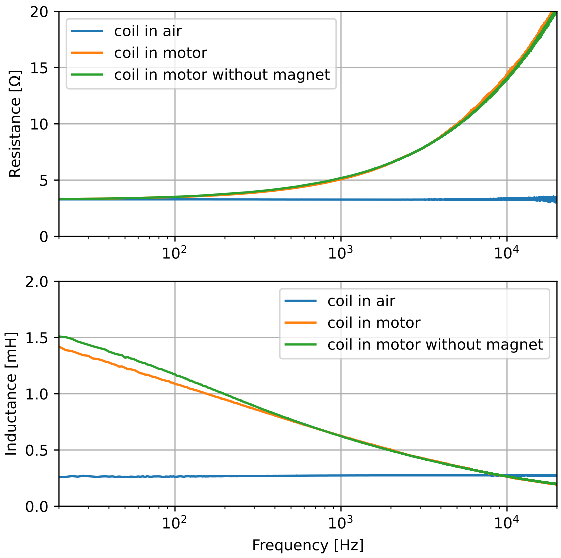


Figure 2: An example of blocked impedance (apparent resistance above, apparent inductance bellow) measured as a function of frequency.

such a measurement, the voice-coil is blocked and cannot move at all. An example of such a measurement on a midrange electrodynamic loudspeaker, taken from [8], is shown in Fig. 2. It shows an important variation of the apparent resistance (Fig. 2 above) and apparent inductance (Fig. 2 below) with frequency (orange curves). The apparent resistance increases with frequency, while the apparent inductance decreases. When the coil is measured in air (blue curves), the apparent resistance and inductance are constant with frequency. Note also that after removing the magnet from the motor, the impedance measurements are almost identical (green curves) to the measurements with the motor.

2.1 Apparent inductance

The apparent inductance exhibits a different behavior when the voice-coil is blocked inside the motor compared to when it is measured in air. At low frequencies, its value is higher when the coil is blocked. With increasing frequency, the apparent inductance decreases (Fig. 2 below). The higher value at low frequencies can be explained by the much higher permeability of the iron core compared to air, as detailed in [8]. The decreasing inductance with frequency is a result of the eddy currents.

The transformer analogy helps to put this decrease in perspective. The primary winding of the transformer represents the voice-coil, while the secondary winding represents the conductive parts of the motor. In such a transformer, the secondary winding generates a magnetic flux that tries to oppose the flux made by the primary winding, leading to reduced net magnetic flux. The inductance of a coil can be understood as the ability to generate magnetic flux in response to the current flowing through it. As the total flux inside the voice-coil is decreased by the opposing flux of the secondary winding, the apparent inductance is perceived as lower. This effect is proportional to the rate of change of the magnetic field, i.e. the frequency of the current flowing through the voice-coil. At very low frequencies, the rate of change is negligible, the induced eddy currents are small, and the opposing flux by the secondary winding is also small.

2.2 Apparent resistance

While the decrease of inductance with frequency is quite intuitive, the increase of the apparent resistance is less so. To understand this behavior, the transformer analogy is even more handy.

The current flowing through the secondary winding depends on the impedance of the secondary winding. Indeed, the creation of the current in the secondary winding, and similarly the creation of eddy currents, is a result of electromagnetic induction, that can be described by Faraday's law

$$u_{\mathrm{emf}} = \oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}, \qquad (1)$$

where the voltage u_{emf} is called the electromotive force, and Φ is the total magnetic flux. We will discuss the middle term later. This voltage is generated by the varying magnetic flux ($d\Phi/dt$) from the primary winding (the voice-coil), thus, it is independent of the surrounding materials. In other words, this voltage exists independently of the secondary winding properties, and exists even if the secondary winding is not present.

The conclusion of the previous paragraph might be perceived as counterintuitive, or even incorrect, and thus requires clarification. The existence of the voltage, even in the absence of the winding, does not mean that there is a measurable voltage in a conventional sense. First, without a physical winding, there are no terminals across which a voltage difference can be measured. Second, even if one would try to put the voltmeter probes there and short them, the measured voltage would depend on the position of the voltmeter wires, because of the path-dependent nature of the induced electric field. Indeed, the voltage u_{emf} is not a potential difference in the sense of Kirchhoff's voltage law which assumes a conservative electric field. The electric field induced by a time-varying magnetic field is non-conservative, meaning that u_{emf} depends on the chosen path of integration. The middle term of Eq. (1) is the line integral of the electric field $\mathbf{E}$ around a closed loop C. In the case of the secondary winding, we choose the path of integration to follow the wire of the winding itself. However, this path conceptually exists even if the secondary winding is not there and consequently the voltage u_{emf} exists as well. This means that if a free electron were placed somewhere along this hypothetical path, it would experience a force due to the induced electric field. On the other hand, the current running in the secondary winding needs the secondary winding to exist. To determine the current, one needs to know the impedance of the secondary winding; it's the simple Ohm's law.

In the case of the loudspeaker motor, we can consider, for the sake of simplicity, that the load of the secondary winding is a simple resistance. This resistance is proportional to the resistivity of the material, usually iron in the case of the loudspeaker motor. Using the transformer analogy, the apparent resistance seen by the voice-coil can be understood as the inner voice-coil resistance connected in series with the resistance of the secondary winding reflected to the primary side.

This explanation, however, seems to lack the key point. The serial combination of two resistances remains constant, which is contrasting with the observed increase of the apparent resistance with frequency. Indeed, we used, up to now, the ideal transformer model, which reflects the impedance of the secondary winding to the primary side independently of the frequency. In such a case, the resistance of the secondary winding is simply reflected to the primary side.

A real transformer, on the other hand, does not work perfectly in the whole frequency range. At very low frequencies, the change of the magnetic flux is negligible, and consequently, the transformer does not transfer energy from one winding to the other. At very low frequencies, the resistance of the secondary winding is not fully reflected to the primary side. This can be easily demonstrated by measuring the direct-component (DC) resistance of the primary winding using an ohmmeter. The measured resistance would be equal to the resistance of the primary winding only, as the secondary winding is not contributing to the total impedance at DC. The same effects happen with the loudspeaker voice-coil and eddy currents. At very low frequencies, the apparent resistance is equal to the resistance of the voice-coil only, as seen in Fig. 2. As the frequency increases, the apparent resistance increases as well, as the resistance of the conductive materials surrounding the voice-coil (the secondary winding) is reflected to the primary side better and better. At very high frequencies, one should observe a similar behavior as with the ideal transformer, i.e. the apparent resistance should be almost equal to the resistance of the voice-coil plus the resistance of the conductive materials surrounding the voice-coil fully reflected to the primary side.

The explanation provided above should help the reader to understand the results of the measured blocked impedance, presented in Fig. 2. However, to support this interpretation, a more formal mathematical derivation is needed. In the following section, we derive the input impedance of a transformer, and we show how the apparent resistance and inductance depend on frequency using a more academic approach.

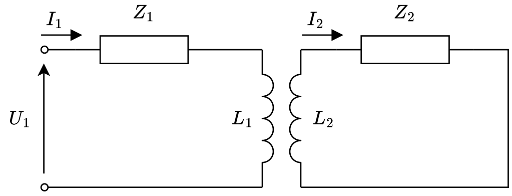


Figure 3: Circuit diagram of the transformer system

3 Tranformer analogy

Let's start with a simple circuit, depicted in Fig. 3, with a transformer consisiting of two windings, one called primary and the other secondary. The self-inductance of the primary winding and secondary winding are denoted as L_1 and L_2 respectively. The impedance connected to the primary winding is Z_1 and the load impedance of the secondary winding is Z_2. These impedances can, for instance, represent the Joule losses in both coils. This circuit is described in the frequency domain as

$$U_1(\omega) = Z_1(\omega)I_1(\omega) + j\omega L_1 I_1(\omega) - Mj\omega I_2, \tag{2}$$
$$0 = Z_2(\omega)I_2(\omega) + j\omega L_2 I_2(\omega) - Mj\omega I_1, \tag{3}$$

where $U_1(\omega)$ is the spectrum of the voltage applied to the primary winding, and $I_1(\omega)$ and $I_2(\omega)$ denote the spectra of the currents in the primary and secondary windings, respectively. The mutual inductance M, between the two coils, is given by

$$M = \kappa\sqrt{L_1 L_2}. \tag{4}$$

where κ is the coupling coefficient, which represents how effectively the magnetic flux links the two coils. Its value ranges between 0 (no coupling) and 1 (perfect coupling).

From Eq. (3) we first solve for

$$I_2(\omega) = \frac{Mj\omega I_1(\omega)}{Z_2(\omega) + j\omega L_2}, \tag{5}$$

then we substitute $I_2(\omega)$ into Eq. (2), to finally express the input impedance as

$$Z(\omega) = Z_1(\omega) + j\omega L_1 + \frac{\kappa^2\omega^2 L_1 L_2}{Z_2(\omega) + j\omega L_2}. \tag{6}$$

We will later use Eq. (6) to derive expressions for the real and imaginary parts of the impedance.

3.1 Ideal transformer model

In an ideal transformer, we consider the following assumptions. First, the coupeling factor κ is equal to 1, meaning that the two windings are perfectly coupled, in other words the magnetic flux created by one winding is fully linked to the other winding. Second, the permeability μ of the core is infinite. Indeed, the permeability of a material represents how easily it allows magnetic flux to pass through it. Setting the permeabilty $\mu \to \infty$ means that all the magnetic flux is perfectly guided through the core without any leakage or opposition (the reluctace of the core is zero).

For now, let's continue simplifying Eq. (6), keeping the coupling factor κ equal to 1, resulting in

$$Z(\omega) = \frac{\big(Z_2(\omega) + j\omega L_2\big)\big(Z_1(\omega) + j\omega L_1\big) + \omega^2 L_1 L_2}{Z_2(\omega) + j\omega L_2} = \\ = \frac{Z_1(\omega)Z_2(\omega) + j\omega L_2 Z_1(\omega) + j\omega L_1 Z_2(\omega)}{Z_2(\omega) + j\omega L_2}. \tag{7}$$

Let's now divide both the numerator and the denominator by $j\omega L_2$, to get

$$Z(\omega) = \frac{\dfrac{Z_1(\omega)Z_2(\omega)}{j\omega L_2} + Z_1(\omega) + \dfrac{L_1}{L_2}Z_2(\omega)}{\dfrac{Z_2(\omega)}{j\omega L_2} + 1}. \tag{8}$$

As the inductance of a coil is proportional to the permeability μ of its core material, and as the permeability is considered infinte in the ideal transformer, both L_1 and L_2 are very high compared to other terms in Eq. (8). As a consequence, using $\frac{1}{j\omega L_2} \to 0$ leads to

$$Z(\omega) \to Z_1(\omega) + \frac{L_1}{L_2}Z_2(\omega). \tag{9}$$

Finally, as the inductance L is proportional to the square of the number of turns N^2 and as both coils are wound on the same core, we can write

$$Z(\omega) = Z_1(\omega) + \left(\frac{N_1}{N_2}\right)^2 Z_2(\omega), \tag{10}$$

where N_1 and N_2 are the number of turns in the primary and secondary coils, respectively.

This shows that in an ideal transformer, the impedance reflects from the secondary to the primary side scaled by the square of the turns ratio. In the case of eddy currents in loudspeaker motors, explained in Section 2, the resistance of the secondary winding (the conductive materials surrounding the voice-coil) is reflected to the primary side, and it is added to the resistance of the primary winding (the voice-coil).

3.2 Real shorted transformer with losses

The real transformer differs from the ideal one in several aspects. The core has finite permeability, meaning that the self-inductances L_1 and L_2 are finite; the coupling factor κ is not equal to one; and also both windings exhibit Joule losses. To take these assumptions into account, we consider Eq. (6) in which $Z_1(\omega) = R_1$, and $Z_2(\omega) = R_2$, represent Joule losses. As the eddy currents are closed loops, we can assume the secondary winding be shorted, leading to

$$\begin{aligned} Z(\omega) &= R_1 + j\omega L_1 + \frac{\kappa^2\omega^2 L_1 L_2}{R_2 + j\omega L_2} = \\ &= R_1 + j\omega L_1 + \frac{\kappa^2\omega^2 L_1 L_2 (R_2 - j\omega L_2)}{(R_2 + j\omega L_2)(R_2 - j\omega L_2)} = \\ &= R_1 + j\omega L_1 + \kappa^2\frac{\omega^2 L_1 L_2 R_2 - j\omega^3 L_1 L_2^2}{R_2^2 + \omega^2 L_2^2} = \\ &= R_1 + j\omega L_1 + \kappa^2\frac{\dfrac{L_1}{L_2}R_2 - j\omega L_1}{\left(\dfrac{R_2}{\omega L_2}\right)^2 + 1}. \end{aligned} \tag{11}$$

Separating the real and imaginary parts, we get the real part (the apparent resistance) as

$$R(\omega) = R_1 + \kappa^2 \frac{L_1}{L_2}\frac{R_2}{\left(\dfrac{R_2}{\omega L_2}\right)^2 + 1}, \tag{12}$$

and the imaginary part divided by ω (to get the apparent inductance) as

$$L(\omega) = L_1 - \kappa^2 \frac{L_1}{\left(\dfrac{R_2}{\omega L_2}\right)^2 + 1} \tag{13}$$

These expressions show that the apparent resistance $R(\omega)$ increases with frequency, whereas the apparent inductance $L(\omega)$ decreases with frequency.

3.3 Simulation of the blocked impedance

Fig. 4 shows the blocked impedance simulation for $R_1 = 4\,\Omega$, $R_2 = 1\,\mathrm{m}\Omega$, $L_1 = 10$ mH, and $L_2 = 1\ \mu$H, turn ratio $N_1 : N_2 = 100 : 1$, and $\kappa = 1$. The apparent resistance $R(\omega)$ (real part of the input impedance $Z(\omega)$) and the apparent inductance $L(\omega)$ (imaginary part of the input impedance $Z(\omega)$ divided by the angular frequency ω) are plotted as a function of frequency for both the transformer with finite permeability (Eq. (8), blue line) and the ideal transformer (Eq. (11), orange dashed line).

Considering an ideal transformer, with the secondary winding being loaded by the resistance R_2, the latter is reflected to the primary side with the square of the turns ratio. The apparent resistance is constant with frequency, and its value corresponds to the ideal transformer (Eq. 11), i.e., $R = R_1 + \left(\frac{N_1}{N_2}\right)^2 R_2$. The apparent inductance is equal to zero as both Z_1 and Z_2 are real (Eq. 11). Both are depicted as orange dashed lines in Fig. 4. For the real transformer with finite permeability, both the apparent resistance and the apparent inductance are frequency-dependent, depicted in Fig. 4 with blue line, and their values are assymptotically approaching the ideal transformer values at high frequencies.

Similar simulation results are shown in Fig. 5 for the same parameters as in Fig. 4, but with a coupling factor $\kappa = 0.9$. The apparent resistance and inductance are plotted as a function of frequency for both the transformer with finite

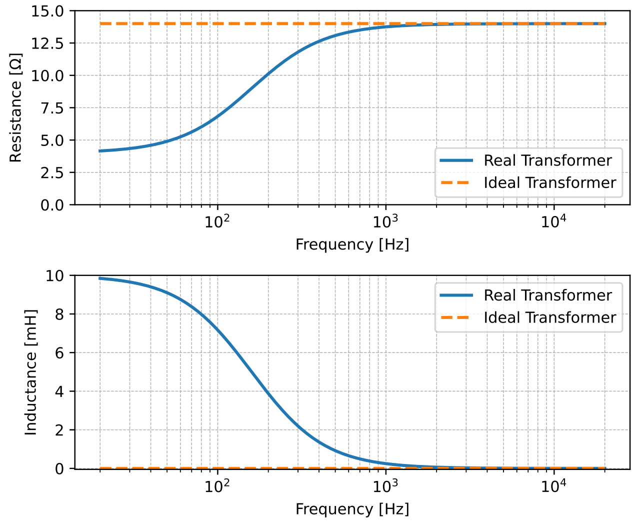


Figure 4: Input impedance (real part above, imaginary part divided by angular frequency below) vs. frequency for both the transformer with finite permeability (blue line) and ideal transformer (orange dashed line). The coupeling factor κ is set to 1.

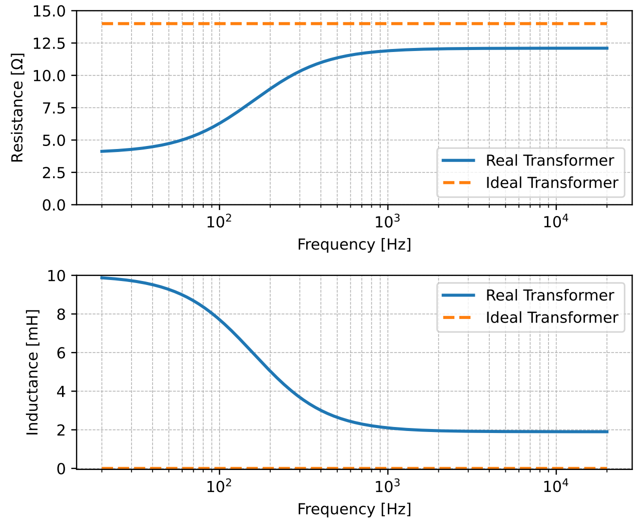


Figure 5: Input impedance (real part above, imaginary part divided by angular frequency below) vs. frequency for both the transformer with finite permeability (blue line) and ideal transformer (orange dashed line). The coupeling factor κ is set to 0.9.

permeability (Eq. (8), blue line) and the ideal transformer (Eq. (11), orange dashed line). The apparent resistance is still increasing with frequency, and the apparent inductance is decreasing with frequency, as in the previous case. The difference between the two cases is that both the apparent resistance and the apparent inductance do not reach the ideal transformer values at high frequencies. This is due to the fact that the coupling factor κ is less than 1, meaning that not all the magnetic flux created by one winding is linked to the other winding.

The results of these simulations are consistent with the theoretical description provided in Section 2. At high frequencies, the blocked impedance of a voice-coil inside a loudspeaker motor behaves similarly to an ideal transformer, where the secondary winding represents the conductive materials surrounding the voice-coil. At very low frequencies, approaching direct current (DC), the rate of change of magnetic flux ($\frac{d\Phi}{dt}$) is negligible. The lack of flux change means that the transformer does not transfer energy between the primary to the secondary windings effectively. At higher frequencies, the rate of change of magnetic flux increases and the transformer becomes better in transferring energy, little by little approaching the ideal transformer behavior.

4 Discussion

The transformer analogy is a powerful tool helping to understand the influence of eddy currents on the voice-coil behavior. However, it is important to note that this analogy is not perfect. The impedance of the secondary winding, i.e., the place where the eddy currents flow, cannot be represented by a simple impedance, like resistance or inductance. As shown in previous works [1], the eddy currents are confined roughly within a skin depth of the conductive material, and their distribution is not uniform. This skin depth is frequency-dependent. Hence, a simple transformer analogy is not sufficient to fully describe the eddy currents in a loudspeaker motor. One possible solution is to represent the skin effect and the skin depth by several layers of conductive materials, each with its own impedance. This approach is similar to the one used in [5] where an infinite ladder network is used to represent the eddy currents.

5 Conclusion

In this paper, we propose a transformer analogy as an intuitive approach to explain the effects of eddy currents in loudspeakers. By considering the voice-coil as the primary winding and the surrounding conductive materials as a shorted secondary winding, we demonstrate how eddy currents influence both the apparent resistance and the apparent inductance of the blocked impedance. This transformer analogy should provide an intuitive pedagogical tool for students and engineers, bringing a clearer understanding of how eddy currents affect the loudspeaker behavior.

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