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# Measurement of loudspeaker parameters: A pedagogical approach

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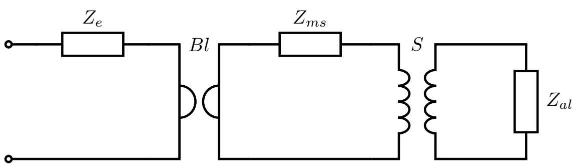
**Abstract**
In this paper, we discuss a pedagogical approach to measuring an electrodynamic loudspeaker and understanding the importance of the development of more precise models. The traditional way of measuring an electrodynamic loudspeaker consists of adjusting the loudspeaker model parameters to get the best fit with the measured signals. The method proposed in this paper is based on a straightforward idea that permits first to estimate the force factor without any model assumption and then to separate the electrical and mechanical impedance intuitively. Consequently, both impedances can be studied separately to highlight the importance of using models incorporating eddy currents on the electrical side and creep effect on the mechanical side. Python code to follow all the steps is available online on https://ant-novak.com/pages/speaker_params/.
Keywords: Loudspeaker measurement, Thielle-Small parameters

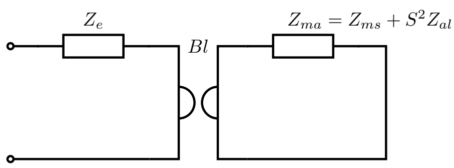
## 1 INTRODUCTION

The basic principle of an electrodynamic loudspeaker, invented almost 100 years ago [9], is well-known thanks to electrical analogies. These analogies have been next used in works of Thiele [13, 14] and Small [12] to model the low-frequency behavior of a loudspeaker thanks to parameters that are measurable. A general loudspeaker model is composed of three parts, the electrical one with electrical impedance $Z_e$, the mechanical part with mechanical impedance $Z_{ms}$ and the acoustical part with load impedance $Z_{al}$. These three parts are joined by the force factor $Bl$ (electro-mechanical) and by an equivalent surface area $S$ (mechano-acoustical) (see Fig. 1a). The acoustical part depends on the radiation (load) impedance of both sides of the membrane and is sometimes included in the mechanical part (Fig. 1b).

In the traditional Thiele-Small model, the electrical part is represented by an R-L circuit in which $R_e$ and $L_e$ represent the voice-coil losses and inductance respectively. An ideal mass–spring–damper system with mass $M_{ms}$, stiffness $K_{ms}$, and mechanical resistance $R_{ms}$ representing the mechanical losses describes the mechanical part. If the acoustical impedance $Z_{al}$ is joined with the mechanical impedance $Z_{ms}$ they form a mechanical impedance $Z_{ma}$. Some works, trying to improve the original lumped-element linear Thiele and Small (TS) model, are presented below.



(a)



(b)

Figure 1. General model of a loudspeaker.

## 1.1 Models of Voice-Coil

The classical loudspeaker motors include iron pieces whose electrical conductivity is relatively high leading to the presence of eddy currents [11, 18]. There have been many attempts to include the effects of eddy currents into the loudspeaker models by modifying the electrical part. Some of them remain simple, keeping the total number of parameters low, others are more precise but for the cost of higher complexity. The model proposed by Thiele [14] put a resistance $R_s$ in parallel with the inductance $L_e$. This idea has been then extended by Murray [6] and later by Klippel [3]. Another approach consist in applying frequency dependent elements, e.g. the models of Vanderkooy [18], Wright [20], Leach [5], Thorborg and Unruh [17], or Brunet and Shafai [1] which uses fractional derivatives.

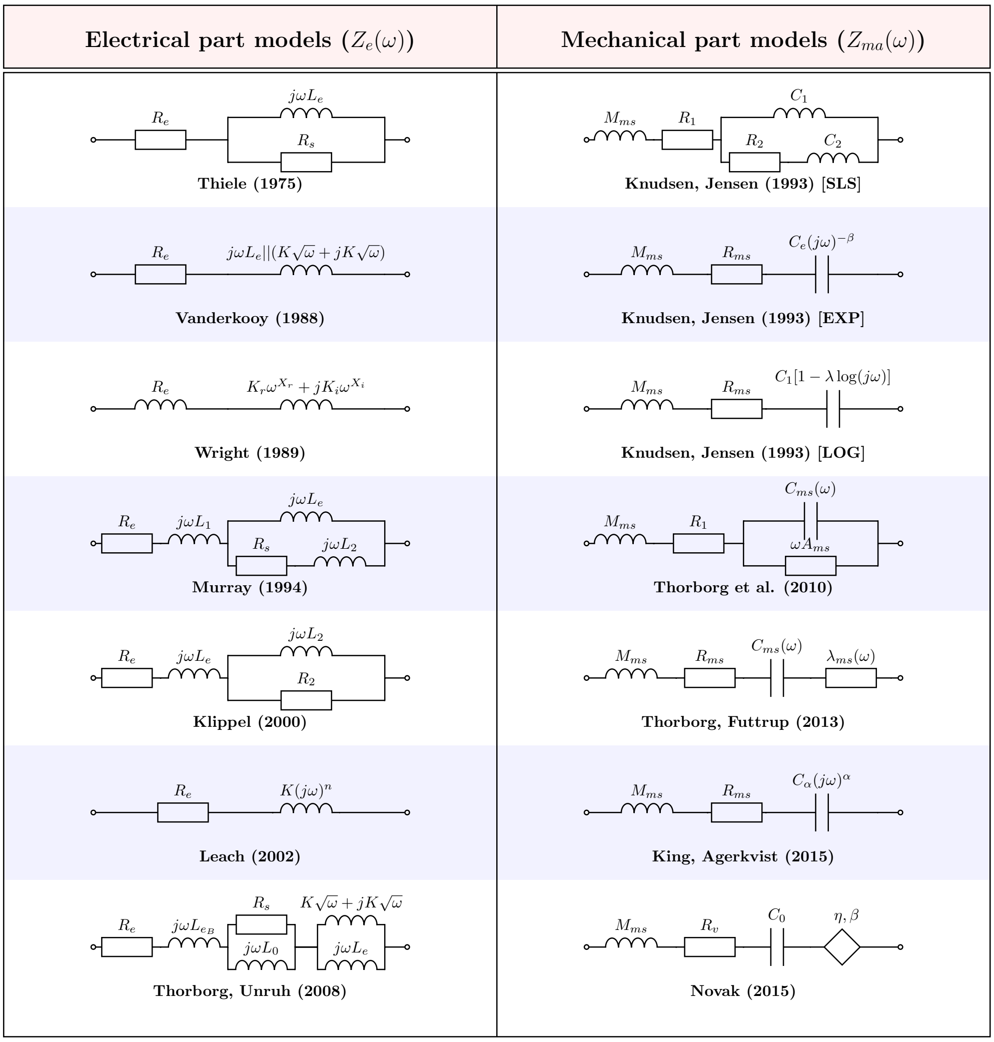



Figure 2. Historical evolution of electrical and mechanical part of loudspeaker models, taking into account eddy currents and viscoelastic effects.

### 1.2 Models of Suspensions

While the electrical part is quite well understood and modeled, the mechanical part of the loudspeaker remains up to now the less understood part of the loudspeaker. Visco-elastic materials used for the surround and the spider are very complex systems not only due to complex material structures but also due to their geometry.

Several studies [4],[16],[10] have shown that the traditional mass–spring–damper model fails to describe the complex viscoelastic behavior of these materials even at low amplitudes. One of the first attempts to take into account the viscoelastic effects is presented in the study of Knudsen and Jensen [4] in which three models are proposed: (a) the Standard Linear Solid (SLS) model, (b) the EXP (exponential) model, and (c) the LOG (logarithmic) model. All three models have shown an improvement over the traditional model; the LOG performing the best. The LOG model is then modified in the works of Thorgborg et al. [16, 15]. King and Agerkvist [2] use the EXP model applying fractional derivatives. Finally, in [7] a fractional element, often used in the field of rheology, is adapted for loudspeaker suspension modeling.

### 1.3 Which Model to Choose?

The number of existing models of voice-coil and surround is quite high and may lead to a source of confusion, especially for students and young engineers. Each model has its pros and its cons, but none of the models describes the loudspeaker behavior perfectly. In general, if we want to have a better model describing loudspeaker behavior more precisely, we need a model with more parameters. On the other hand, in many applications, the simplicity (low number of parameters) wins over the model precision.

Side by side with the development of new models there are many methods for parameters estimation associated with the models. These methods are usually based on fitting algorithms and least square error (LSE) minimization that is applied to the measured data. Such an approach have two notable drawbacks. First, if a part of the model does not fit the reality correctly, e.g., a loudspeaker with a huge creep effect modeled using a classical TS model, the estimated parameters (including the electrical ones) are all in error. Second, fitting all the parameters at once does not provide any information from which part (i.e., voice-coil, magnetic circuit, suspension) possible errors come from.

The following part of the paper proposes a measurement method based on an intuitive approach that also answers many questions one can ask regarding the choice of a model and the loudspeaker behavior.

## 2 PROPOSED METHOD

### 2.1 Prerequisites

In the following, we consider that the loudspeaker under test has been mounted in an "almost-infinite" baffle and measured using any wide-band signal and that the voltage, current, and velocity signals have been acquired. In the case of the example presented in this paper, we used a multi-tone signal consisting of 648 frequencies $f$ logarithmically spaced between 10 Hz and 20 kHz, forming an angular frequency vector $\omega = 2\pi f$ with 648 points. Next we suppose that the frequency-domain quantities, i.e. voltage $U(\omega)$, current $I(\omega)$, and velocity $V(\omega)$ are calculated from the measured signals (see Table reftab:table1). For frequencies above 4 kHz, the

Table 1. Measured and derived quantities used in the paper ($Bl$ being the loudspeaker force factor).

| | voltage | current | velocity | input impedance | electrical impedance | mechanical impedance |
|---|---|---|---|---|---|---|
| time-domain | $u(t)$ | $i(t)$ | $v(t)$ | — | — | —- |
| frequency-domain | $U(\omega)$ | $I(\omega)$ | $V(\omega)$ | $Z_{in}(\omega) = \dfrac{U(\omega)}{I(\omega)}$ | $Z_e(\omega) = Z_{in}(\omega) - Bl\,V(\omega)$ | $Z_m(\omega) = \dfrac{Bl\,I(\omega)}{V(\omega)}$ |

vibrometer values $V(\omega)$ are set to zero to ignore high-frequency modal behavior. The data file and the Python code with all the following steps are available online (`https://ant-novak.com/pages/speaker_params/`) [8].

### 2.2 Bl estimation

The input impedance $Z_{in}(\omega)$ of the loudspeaker estimated as the ratio between the measured voltage $U(\omega)$ and current $I(\omega)$ (see Table 1), depicted in Fig. 3 for the measured data, is given as

$$Z_{in}(\omega) = Z_e(\omega) + Bl\frac{V(\omega)}{I(\omega)}, \tag{1}$$

where $Bl$ is the force factor of the loudspeaker and $Z_e(\omega)$ is the electrical impedance of the voice coil, which can be expressed as

$$Z_e(\omega) = Z_{in}(\omega) - Bl\frac{V(\omega)}{I(\omega)}. \tag{2}$$

The main idea of the estimation of the force factor $Bl$ independently of other model parameters (such as those in Fig. 2) is that the voice-coil impedance $Z_e(\omega)$ has no reason to exhibit any resonance at low frequencies. The resonance behavior is due to the mechanical part, in other words, due to the term $Bl\frac{V(\omega)}{I(\omega)}$. Consequently, we can evaluate Eq. (2) for different estimated values of $\overline{Bl}$. Such value $\overline{Bl}$ that minimizes the resonant behavior of $Z_e(\omega)$ is then selected as the $Bl$ value of the loudspeaker. This process is graphically represented in Fig. 4. In this particular example the $Bl = 4.8$ Tm.

### 2.3 Electrical Impedance

Once the value of $Bl$ is estimated, we can separate the electrical impedance $Z_e(\omega)$ using Eq. (2). The Thiele-Small model predicts the electrical impedance behavior being influenced by two constant parameters, resistance $R_e$ and inductance $L_e$. The resistance is the real part of $Z_e(\omega)$, and the inductance is the imaginary part $Z_e(\omega)$ divided by the angular frequency $\omega$. These two quantities are deduced from the real data from the measurement and depicted in Fig. 5. It is clear from both figures that both quantities are highly dependent on frequency contrary to the Thiele-Small model in which both quantities are constant. For this reason they are called "apparent resistance" $R_e(\omega)$ and "apparent inductance" $L_e(\omega)$.

The reason for such a variation with frequency is the presence of eddy currents in the iron pole pieces. They increase the apparent resistance due to the thermal effects of eddy currents passing inside the iron and decrease the apparent inductance by mutual inductance. The eddy currents depend on frequency due to the skin effect [18].

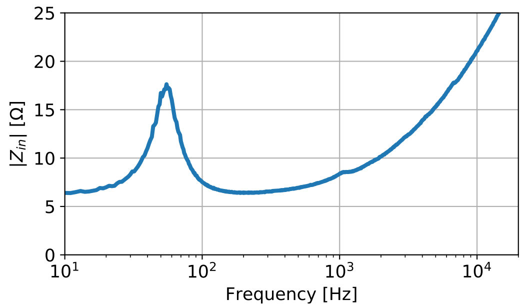



Figure 3. Input impedance of the loudspeaker estimated from the measured voltage and current.

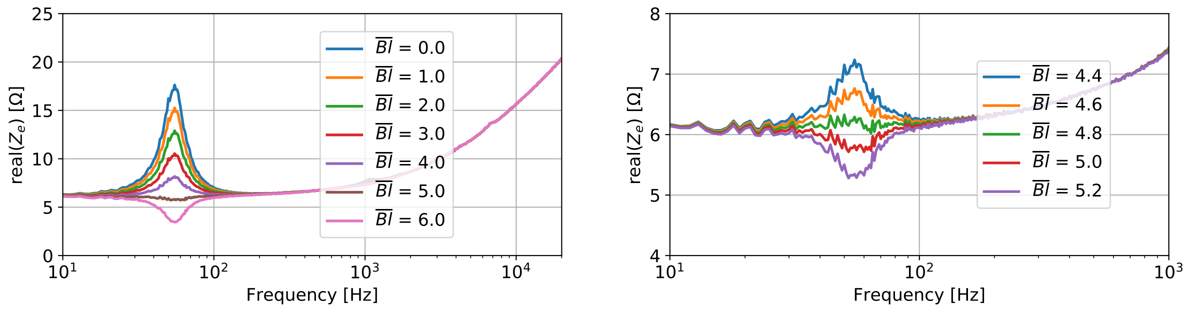
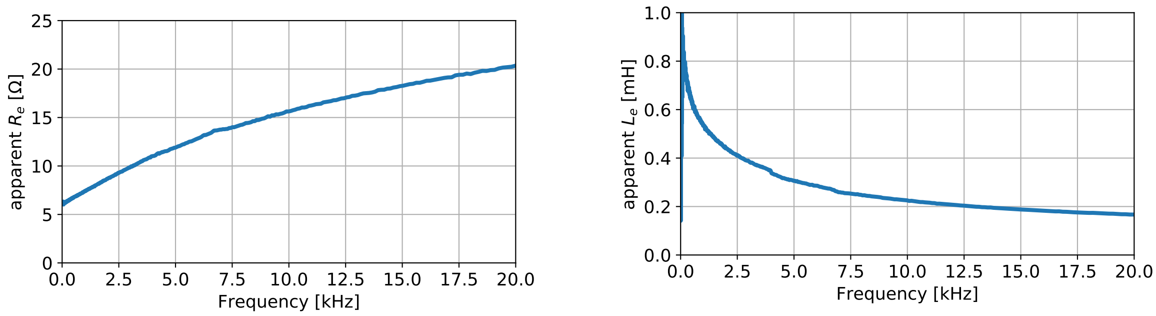



Figure 4. Graphical representation of the force factor $Bl$ estimation from the measured data. The value of $\overline{Bl}$ that removes the resonance from the electric impedance $Z_e(\omega)$ is selected as the best estimate of $Bl$.




(a) $\mathrm{real}\left\{Z_e(\omega)\right\}$ (b) $\mathrm{imag}\left\{\frac{Z_e(\omega)}{\omega}\right\}$

Figure 5. Apparent resistance and inductance of the loudspeaker voice-coil, both depending on frequency, estimated from the measured data.

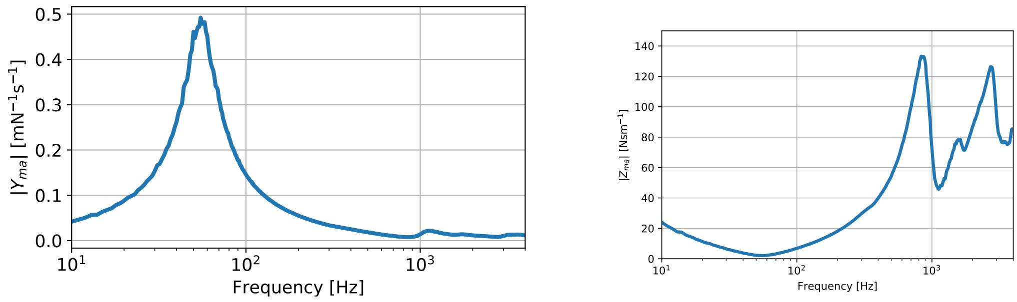
The effect of eddy currents can now be included in the model of the loudspeaker by choosing any of the models, e.g., one of those presented in Table 1. The model fitting can be done only on the isolated electrical part $Z_e(\omega)$ with no influence on other parameters of the loudspeaker.

### 2.4 Mechanical Impedance

In the next step, we can estimate the mechanical impedance $Z_m(\omega)$ that includes also the acoustical radiation impedance. The mechanical impedance $Z_m(\omega)$ (or mechanical admittance $Y_m(\omega) = Z_m^{-1}(\omega)$) can be deduced as

$$Z_m(\omega) = Bl\frac{F(\omega)}{V(\omega)}, \tag{3}$$

In Fig. 6 both the mechanical admittance and mechanical impedance are depicted as a function of frequency. By observing both impedance curves, we can note two distinct parts. At low frequencies (up to 300 Hz) the behavior is driven by a damped mass-spring system with a resonance frequency near 55 Hz. At high frequencies (above 300 Hz) the behavior starts to become less predictable (see Fig. 6b) due to a non-piston motion of the mechanical part (break-up, modal behavior). Since the modal behavior is out of the scope of this paper and since we consider only the piston motion, we use only the data from low frequencies (between 20 Hz and 250 Hz) for further analysis of mechanical part.




(a) mechanical admittance

(b) mechanical impedance

Figure 6. The absolute value of the mechanical admittance (a) and the mechanical impedance (b) as a function of frequency, estimated from the measured data.

### Mechanical Resistance

The Thiele-Small model considers the mechanical impedance to behave like a mass-spring-damper system with mass $M_{ma}$, stiffness $K_{ma}$, and mechanical resistance $R_{ma}$ related to the mechanical impedance as

$$Z_m(\omega) = j\omega M_{ma} + R_{ma} + \frac{K_{ma}}{j\omega}. \tag{4}$$

The real part of the mechanical impedance $Z_m(\omega)$ represents mechanical losses. According to Eq. (4), the mechanical resistance is constant and equal to $R_{ma}$. The real part of the mechanical impedance $Z_m(\omega)$ depicted in Fig. 7 for the measured data shows an important frequency dependency with frequency and is called "apparent mechanical resistance". Indeed, this kind of behavior is related to a creep effect [4, 7] and can be taken into account by applying a model that includes these effects (see Table 1).

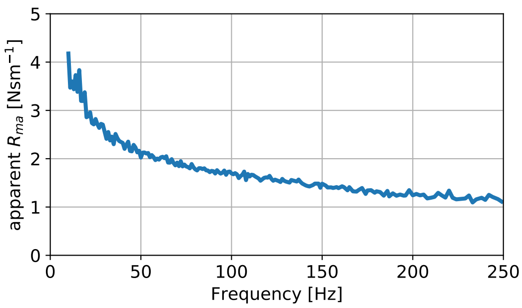


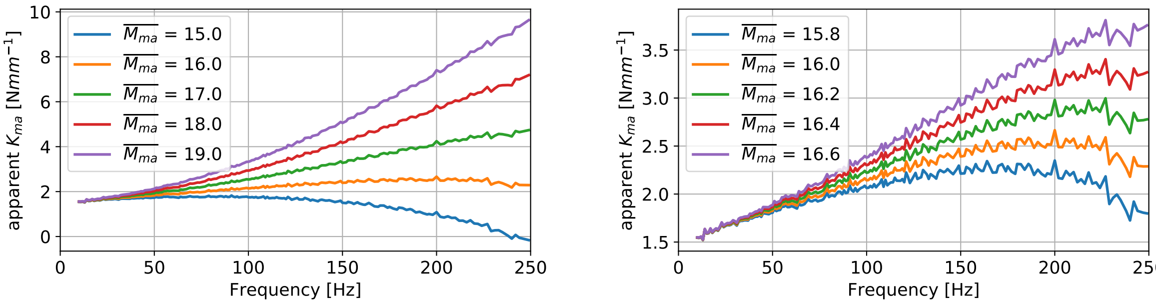

Figure 7. The real part of the mechanical impedance representing the mechanical losses, estimated from the measured data.

### Moving Mass and Stiffness

Finally, the imaginary part of the mechanical impedance $Z_m(\omega)$ represents the mechanical energy storage. According to the Thiele-Small model, the stiffness $K_{ma}$ is related to the moving mass $M_{ma}$ as

$$K_{ma} = \omega^2 M_{ma} - \text{imag}\left\{Z_m(\omega)\right\}\omega. \tag{5}$$

To find both, one can fit the imaginary part of the mechanical impedance to the measured data. However, it is known that the stiffness is frequency dependent due to the creep effect and that the moving mass can be frequency dependent as well due to the inertial air load [19]. Nevertheless, at very low frequencies the moving mass should not vary with frequency a lot [19]. We consider it to be constant in the selected frequency region from 20 Hz to 250 Hz and we apply a similar technique as we used for the force factor estimation by plotting the apparent stiffness $K_{ma}(\omega)$ calculated using Eq. (5) for several estimated mass $\overline{M_{ma}}$.




(a) (b)

Figure 8. Estimation of the apparent stiffness $K_{ma}(\omega)$ and the mass $M_{ma}$ from the measured data.

Fig. 8 shows the apparent stiffness $K_{ma}(\omega)$ for different values of $\overline{M_{ma}}$. The rough estimation (Fig. 8a) with $\overline{M_{ma}}$ varying from 15 to 19 grams show that there is not a single flat $K_{ma}(\omega)$ curve. For low estimates of $\overline{M_{ma}}$ (e.g., 15 g) the apparent stiffness drops quickly to negative values. On the other hand for high estimates of $\overline{M_{ma}}$ (e.g., 19 g) the apparent stiffness raises with frequency to very high values. A detailed variation of the mass $\overline{M_{ma}}$ (Fig. 8b) shows that it is indeed very difficult to select a single value of $\overline{M_{ma}}$ and one associated curve of $K_{ma}(\omega)$. It is clear that the $K_{ma}(\omega)$ is rising with frequency, which is a well-known phenomena due to the creep effect [4, 7], but for more detailed analysis one would need to chose one of the models from Table 1 and fit the parameters to the data.

## 3 Conclusion

This paper shows that the measurement of linear loudspeaker parameters can be made more intuitively and pedagogically than just by blindly fitting the model to measured data. The estimation consists of several consecutive steps. First, we estimate the force factor *Bl* independently of the chosen model. Then, we separate the electrical and the mechanical impedance, and we show that the classical Thiele-Small parameters that are considered constant vary with frequency. These steps may help to understand why there is a large amount of loudspeaker models available in the literature and their relation to physics. We also show that the mechanical part can be estimated only in the low-frequency region to avoid break-up and modal behavior influence while the electrical part can be estimated in a full frequency region.

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