NON-LINEAR IDENTIFICATION OF AN ELECTRIC GUITAR PICKUP

Antonin NOVAK, Leo GUADAGNIN, Bertrand LIHOREAU, Pierrick LOTTON, Emmanuel BRASSSEUR, Laurent SIMON

Laboratoire d'Acoustique de l'Université du Maine,
UMR CNRS 6613, Le Mans, France
antonin.novak@univ-lemans.fr

ABSTRACT

Physical models of electric guitars are still not very widespread in the scientific literature. Especially, the description of the non linear behavior of pickups still requires some refinements. This paper deals with the identification of pickup non linearities based on a Hammerstein representation, by means of a specific experimental set-up to drive the pickup in a controlled way. A comparison with experimental results shows that the model succeeds in describing the pickup when used in realistic conditions.

1. INTRODUCTION

An electric guitar pickup is a sensor that captures the string vibrations and translates them into an electric signal. It is basically composed of a set of permanent magnets surrounded by an electric coil. A ferromagnetic string vibrating in the vicinity of the pickup results in a variation of the magnetic flux through the coil. According to the Faraday's law, an electrical voltage is then induced across the coil.

A few models of pickup are available in the literature. Some of them are based on integral equations leading to the variation of magnetic flux at the coil location [1]. These models principally show, first, that vertical oscillations of the guitar strings produce a stronger effect than horizontal ones and, second, that there is a noticeable distortion of the electric signal generated by both oscillations. An overview of the modeling issues related to magnetic pickups is available in [2]. It concerns effects of both pickup position and pickup width on the pickup timbre, as well as the effect of the pickup internal impedance. In [2], the magneto-electric conversion done by the pickup is modeled using static non-linearity followed by a simple derivative (Fig. 1). The static non-linearity represents the non-linear relation between the string displacement and the magnetic flux which can be evaluated using computer simulations and implemented as an exponential or N-th order polynomial [3].

On the other hand, studies on non-linear modeling have led to many nonparametric non-linear models. Among these non-linear models, the Volterra series representation is usually considered as an effective one. Nevertheless, it lays down the calculation of multidimensional kernels and in practice, most applications are limited to the second or the third order. Simplified Volterra-based models, namely Hammerstein model (static nonlinear function followed by a linear filter) or Wiener model (linear filter followed by a static nonlinear function) [4], are then often preferred in the case of open-loop systems because of their simpler structure and lower computational cost. Furthermore, for a better accuracy of the estimation, these simple models can be extended to so-called generalized models, such as the generalized Hammerstein model, as shown in Fig. 2. This generalized Hammerstein model is made up of N parallel branches, with each branch consisting of a linear filter $G_n(f)$ preceded by an N-th order power static non-linear function, for $n = 1, N$, and has been successfully tested in [5, 6].

The goal of this paper is to proceed with the identification of pickup linearities based on a generalized Hammerstein representation of the pickup. For this purpose, a specific experimental set-up is used to drive the pickup in a controlled way, and a technique is carried out to get rid of non-linearities due to the driver. One of the aims of this study is to find out if it is meaningful, or not, to use a simple Hammerstein structure given in Fig. 1, as it usually done in modeling the pick-up nonlinearities [1, 2, 3], or if more complex model such as the Generalized Hammerstein one is necessary. The answer is given through the measurement provided in sections 2 and 3 and a comparison between theoretical and experimental results in the case of a realistic use of the pickup is given in section 4.

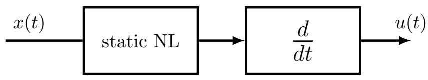


Figure 1: *Non-linear system usually used to model non-linearities of a guitar pickup [2].*

2. MEASUREMENT OF THE PICK-UP NON-LINEARITIES

The first goal of this paper is to identify the pickup in terms of Generalized Hammerstein model (Fig. 2). The output of such a model is governed by the following equation

$$u(t) = \sum_{n=1}^{N} x(t)^n * g_n(t), \tag{1}$$

where $g_n(t)$ is the inverse Fourier transform of $G_n(f)$ and where $*$ stands for convolution.

Since the pickup is an electromagnetic transducer that converts string vibration into an electrical output signal, its experimental characterization is not straightforward. Usually, when measuring a linear or a non-linear device, excitation signal is a controlled one (impulses, swept-sine, pseudo-random sequences) so that output signal can be used to identify the system in terms of a frequency response function (FRF) for a linear system, or in terms of a set of describing functions when dealing with a non-linear system. For a pickup, the excitation signal is the displacement of a plucked string exhibiting a multi-modal and non stationary behavior. Such an excitation is useful for a study in real conditions [7] but can hardly be used to get a FRF or to identify non linearities.

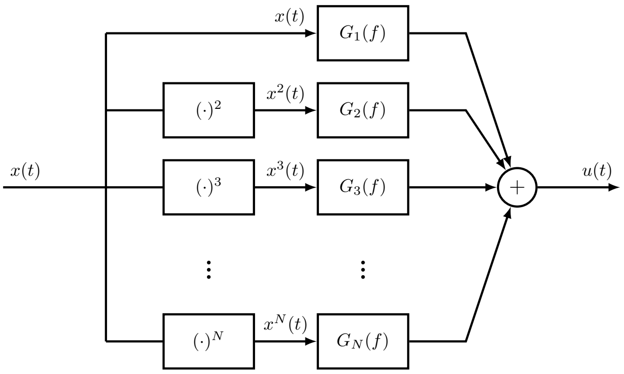


Figure 2: *Generalized Hammerstein model for identifying the non-linearities of the pickup; $x(t)$ and $u(t)$ represent the displacement of the guitar string and the output voltage of the pickup, respectively.*

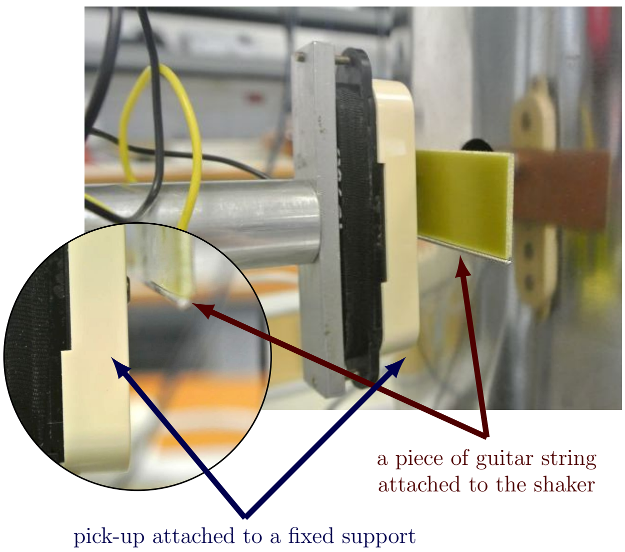


Figure 3: *Measurement device used to characterize the non-linearities of the pickup. The string portion is attached to a shaker; the pickup is attached on a fixed support.*

To control the string displacement, we use the system shown in Fig. 3. A piece of steel string (diameter 1.42 mm) is fixed on a non-magnetic support, itself fixed to a shaker which imposes a string displacement perpendicular to the pickup [8]. The pickup under test is set on a precision movement device which allows adjusting the distance at rest d_0 between the string and the magnet. For this experiment, d_0 is set to $d_0 = 5\ mm$ corresponding approximately to the distance set on a real guitar.

The shaker is driven by a Synchronized Swept Sine signal [9] so that the non-linearities of the whole system, that is the shaker and the pickup in series, can be easily identified using a Generalized Hammerstein model [5].

The synchronized swept-sine is generated using [9]

$$x(t) = \sin\left[2\pi f_1 L \exp\left(\frac{t}{L}\right)\right]. \quad (2)$$

where

$$L = \frac{T}{\ln\left(\frac{f_2}{f_1}\right)}, \quad (3)$$

and where f_1 and f_2 are initial and final frequency respectively and T is duration of the swept-sine. Note that the definition of the exponential swept-sine (Eq. (2)) does not contain the "-1" term contrary to the usual definition [10]. For more details about why the term "-1" should not appear in the exponential swept-sine definition please refer to [9].

To protect the shaker from a possible destruction due to excessive displacement or current, the frequency range is furthermore limited to the span 15 Hz - 500 Hz. The excitation signal is pre-filtered using a linear filter so as to obtain a displacement whose amplitude is almost constant over the frequency span. The peak amplitude is set here to 1 mm. The displacement of the string portion (that is the displacement of the shaker) is measured by means of a vibrometer pointing at the string. The electrical output of the pickup is then connected to an acquisition card which exhibits a high input impedance ($470\ k\Omega$). Consequently, the measured output voltage corresponds to the open-circuit voltage which does not take into account the effect of pickup output impedance.

The Higher Harmonic Frequency Responses (HHFRs) of both the string displacement and the pickup output voltage calculated using the Synchronized Swept Sine method [9] are given in Fig. 4. The method consists in de-convolving the measured signals with a so-called inverse filter as

$$h(t) = \mathcal{F}^{-1}\left[\mathcal{F}[y(t)]\tilde{X}(f)\right], \quad (4)$$

where $y(t)$ is the acquired response of the nonlinear system (displacement or voltage signal) to the synchronized swept-sine, and where the Fourier transform of inverse filter $\tilde{X}(f)$ is given analytically as

$$\tilde{X}(f) = 2\sqrt{\frac{f}{L}} \exp\left\{-j2\pi f L\left[1 - \ln\left(\frac{f}{f_1}\right)\right] + j\frac{\pi}{4}\right\}. \quad (5)$$

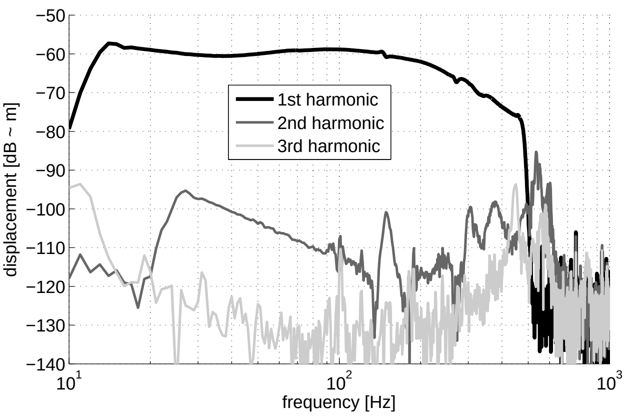
The impulse response $h(t)$ then consists of time-delayed higher harmonic impulse responses, separated by time delays

$$\Delta t_n = L \ln(n), \quad (6)$$

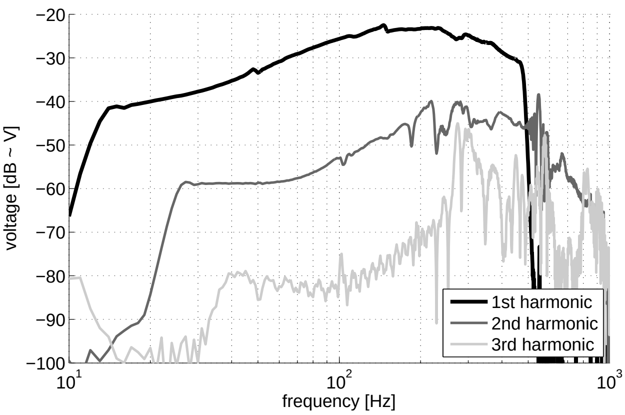
that can be windowed and represented in the frequency domain as The Higher Harmonic Frequency Responses (HHFRs). For more details see [9].

The fundamental harmonic of the string displacement (Fig. 4a) is not flat despite the pre-filtering and the second harmonic reaches -40 dB relative to fundamental harmonic. It is thus rather difficult, or almost impossible, to estimate which part of the HHFRs of the pickup output voltage (Fig. 4b) is due to the pickup behavior and which part is due to the shaker behavior.

To overcome this problem, we use a technique detailed in [11] in which a non-linear system can be identified even if it is preceded by another unknown non-linear system. According to this technique, the non-linear system under test is then described by an N-th order Generalized Hammerstein model, as shown in Fig. 2. For the pickup under test, the magnitude values of the estimated linear filter $G_n(f)$ of the Generalized Hammerstein model are depicted in Fig. 5.



(a) HHFRs of displacement of the string excited by the shaker



(b) HHFRs of the output voltage of the pickup

Figure 4: *Higher Harmonic Frequency Responses (HHFRs) of (a) displacement of the string excited by the shaker and (b) the output voltage of the pickup.*

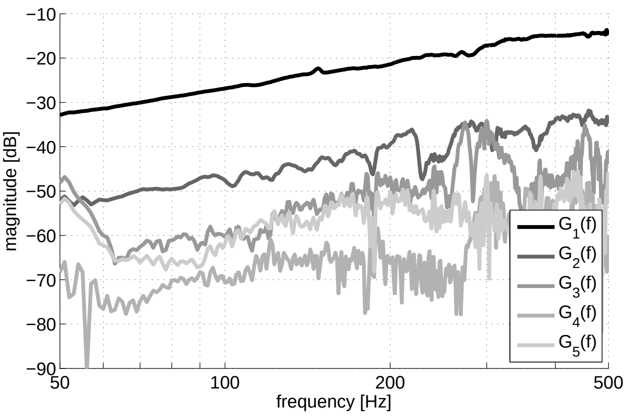
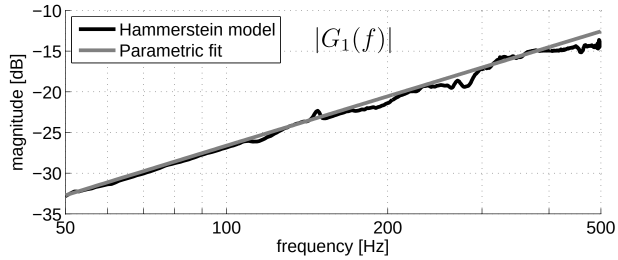
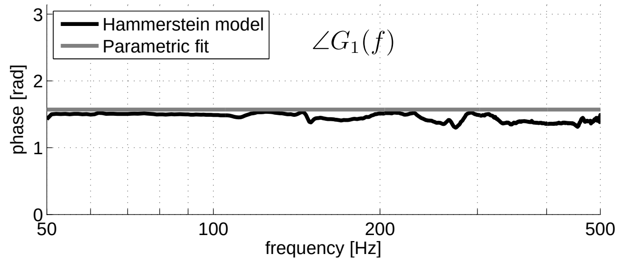


Figure 5: *Magnitude values of the estimated filters $G_n(f)$ of the Generalized Hammerstein model (Fig. 2) of the pickup.*

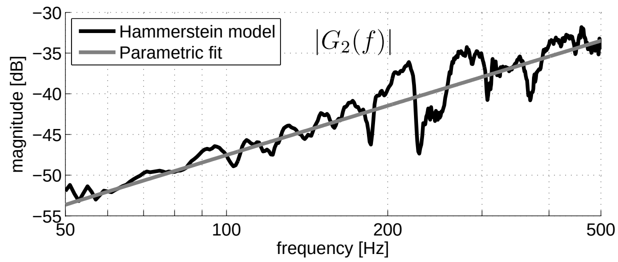


(a)

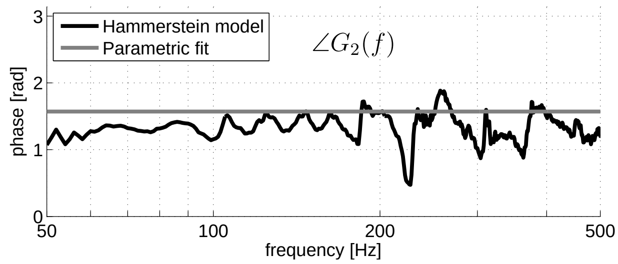


(b)

Figure 6: *Modulus (a) and phase (b) of the first filter $G_1(f)$ of the identified Generalized Hammerstein model of the pickup under test and its parametric fit.*



(a)



(b)

Figure 7: Modulus (a) and phase (b) of the second filter $G_2(f)$ of the identified Generalized Hammerstein model of the pickup under test and its parametric fit.

Table 1: *Coefficients α_n of the estimated parametric Hammerstein model.*

α_1	7.50e−02
α_2	6.75e−03
α_3	2.11e−03
α_4	4.75e−04
α_5	8.31e−04

3. NON-LINEAR PARAMETRIC MODEL OF THE PICKUP

Observing the estimated filters of the Generalized Hammerstein model depicted in Fig. 5, one can note that the dependence on frequency for all filters is approximately 6 dB/oct. Such a slope corresponds to $j2\pi f$ in the frequency domain or to a simple derivative d/dt in the time domain.

It is thus tempting to fit all the filter responses $G_n(f)$ with a function $\alpha_n j2\pi f$ in order to replace each branch of the Generalized Hammerstein model by a multiplicative coefficient α_n in series with a derivative function d/dt. A fit of the first two filters $G_1(f)$ and $G_2(f)$ (magnitude and phase) is depicted in Figs. 6 and 7. It is interesting to note that the estimated phases of both filters are very close to $\pi/2$ within the whole frequency range.

The estimated Generalized Hammerstein model can thus be parametrized and simplified to the following relation

$$u(t) = \sum_{n=1}^{N} \alpha_n \frac{d\, x(t)^n}{dt} = \frac{d}{dt}\left(\sum_{n=1}^{N} \alpha_n x(t)^n\right). \qquad (7)$$

This relation being a time derivative of a Taylor series, we can simplify the Generalized Hammerstein model to a Hammerstein model consisting of a static non-linearity followed by a linear filter (Fig. 1). In this particular case, the static non-linearity is represented by a simple Taylor series with coefficients α_n and the linear filter is represented by a time domain derivative, as shown in Fig. 1. This result tends to confirm the model previously proposed by [2]. The fitted coefficients α_n for the pickup under test are given in Table 1 and the corresponding input-output characteristic is depicted in Fig. 8.

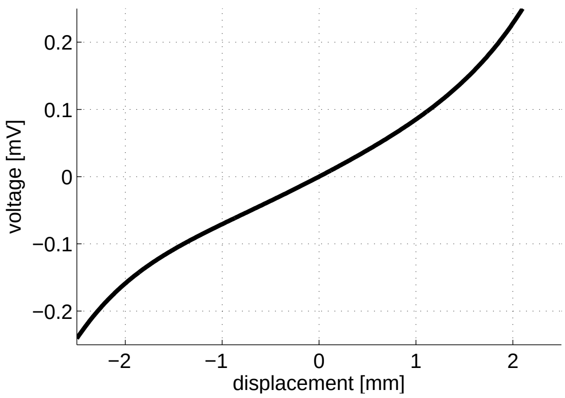


Figure 8: *Input-output graph of the static nonlinerity based calculated as a power series developpement with coefficinets α_n given in Table 1.*

Figure 9: *Picture of the second experiment in which the pickup is placed under a vibrating string.*

4. MODEL VS. REAL GUITAR-STRING SIGNAL

To test the validity of the identified Hammerstein model, we set up a different experiment corresponding to a realistic use of the pickup. For that purpose, we use a lab guitar prototype [8]. A guitar string is fixed on an wooden beam. The string is tuned as open low E (f_0 = 82 Hz). The pickup under test is set on a mechanical arm on which some precision movement pieces are fixed. Thanks to this system, the pickup position under the string can be adjusted along the 3 axes. For this experiment, the pickup is set at 1/4 of the total length of the string corresponding approximately to the neck position on a real guitar and the distance at rest between the string and the pickup is set at 5 mm. A detail of the experiment is shown in Fig. 9. The string is struck using an impact hammer. A vibrometer pointing at the string at the pickup location allows the measurement of the string displacement in the vertical plane. Temporal evolution of both string displacement and pickup output voltage are recorded simultaneously and depicted in Figs. 10 and 11. A zoom on a few periods of both experimental signals is given for three different time lags along the time-varying response. As expected, the string displacement is distorted just after the excitation. It becomes less and less distorted as the harmonics of higher orders fade with time. The output signal of the pickup exhibits the same kind of behavior with time. One can notice that the output voltage is more distorted due to pickup non-linearities.

The displacement signal measured with the vibrometer is then used as the input of estimated parametric Hammerstein model of the pickup and the both measured and synthesized pickup outputs are compared on the same graph (Fig. 11). The difference between estimated and experimental signals is plotted on Fig. 11, showing that the model succeeds in describing the non linear behavior of the pickup when used in realistic conditions.

5. DISCUSSION

The results presented in this paper shows that a simple Hammerstein model seems to be sufficient for the pick-up modeling and that using a Generalized Hammerstein model is not necessary. However, several hypotheses have been put forward simplifying the problem that might be at the origin of small differences between the measured and modeled pick-up outputs compared in Figs. 10

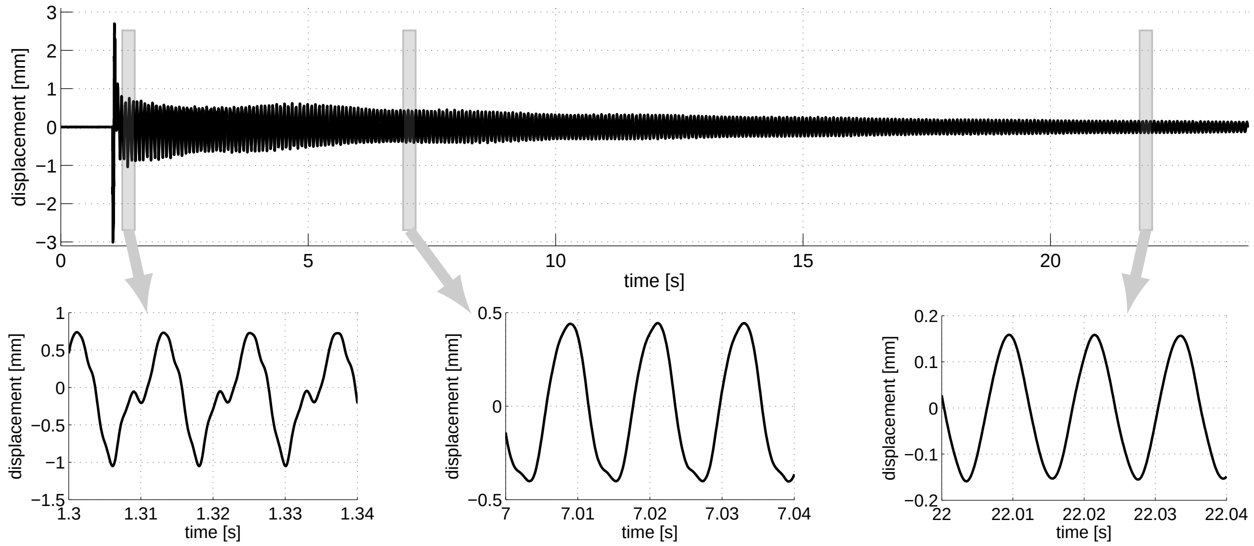


Figure 10: Recorded signals of the vibrating string.

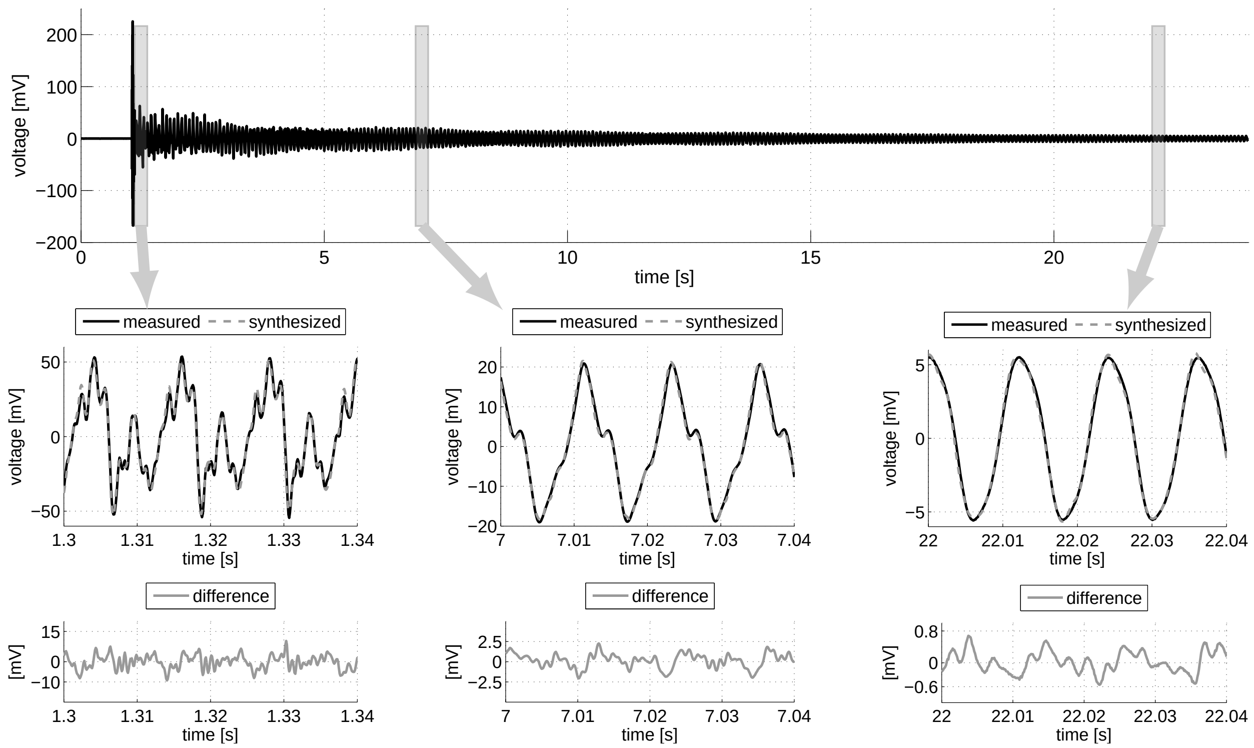


Figure 11: Recorded and synthesized signals of the voltage from the pickup.

and 11.

First, the frequency range of the excitation signal is bellow 500 Hz due to the capacities of the shaker. Using larger frequency range might have been beneficial. The nonlinearities of the pick-up may differ at higher frequencies, and thus, in such a case, a complete Hammerstein Generalized model might be useful. A supplementary study would be necessary to draw a meaningful conclusion.

Next, the movement of the rigid string attached to the shaker exhibits only z-axis polarization, whereas it is known that the string being played by a guitar player exhibits rather an ellipsoid type motion in both y and z directions [12, 13]. A hammer-like impact has been used to excite the string in order to approach the z-axis motion of the string in the comparative measurements whose results are provided in Figs. 10 and 11. However, the real-word pickup behavior might be influenced by 2D movement of the string, even though it is known that the y-axis contribution is rather negligible [14, 8]. Moreover, the piece of the rigid string attached to the shaker is of finite length and does not exhibit any deformation compared to a string attached on guitar.

Even though these phenomena have been neglected, the results presented in this paper show a very good agreement between the output predicted by the model and the output obtained from the experimental measurement.

6. CONCLUSIONS

This paper presents a parametric model of guitar pickup whose parameters are directly estimated experimentally. The validity of this model has been verified for a pickup operating in a realistic way. In future work, this model can be used to synthesize different kinds of existing pickups (single coil pickups, humbuckers). It can also be extended to the synthesis of augmented pickups by artificially modifying the parameters of the model.

7. REFERENCES

[1] Nicholas G Horton and Thomas R Moore, "Modeling the magnetic pickup of an electric guitar," *American Journal of Physics*, vol. 77, no. 2, pp. 144–150, 2009.

[2] Rafael CD Paiva, Jyri Pakarinen, and Vesa Välimäki, "Acoustics and modeling of pickups," *Journal of the Audio Engineering Society*, vol. 60, no. 10, pp. 768–782, 2012.

[3] Luca Remaggi, Leonardo Gabrielli, R de Paiva, Vesa Välimäki, and Stefano Squartini, "A pickup model for the clavinet," in *Digital Audio Effects Conference 2012 (DAFx-12)*, 2012.

[4] Martin Schetzen, "The volterra and wiener theories of nonlinear systems," 1980.

[5] Antonin Novak, Laurent Simon, František Kadlec, and Pierrick Lotton, "Nonlinear system identification using exponential swept-sine signal," *Instrumentation and Measurement, IEEE Transactions on*, vol. 59, no. 8, pp. 2220–2229, 2010.

[6] Marc Rébillat, Romain Hennequin, Etienne Corteel, and Brian FG Katz, "Identification of cascade of hammerstein models for the description of nonlinearities in vibrating devices," *Journal of Sound and Vibration*, vol. 330, no. 5, pp. 1018–1038, 2011.

[7] Mirko Mustonen, Dmitri Kartofelev, Anatoli Stulov, and Vesa Välimäki, "Experimental verification of pickup nonlinearity," in *Proceedings International Symposium on Musical Acoustics (ISMA 2014), Le Mans, France*, 2014, vol. 1.

[8] Pierrick Lotton, Bertrand Lihoreau, and Emmanuel Brasseur, "Experimental study of a guitar pickup," in *Proceedings International Symposium on Musical Acoustics (ISMA 2014), Le Mans, France*, 2014, vol. 1.

[9] Antonin Novak, Pierrick Lotton, and Laurent Simon, "Synchronized swept-sine: Theory, application, and implementation," *Journal of the Audio Engineering Society*, vol. 63, no. 10, pp. 786–798, 2015.

[10] Angelo Farina, Alberto Bellini, and Enrico Armelloni, "Nonlinear convolution: A new approach for the auralization of distorting systems," in *AES 110th convention*, Amsterdam, May 2001.

[11] Antonin Novak, Balbine Maillou, Pierrick Lotton, and Laurent Simon, "Nonparametric identification of nonlinear systems in series," *Instrumentation and Measurement, IEEE Transactions on*, vol. 63, no. 8, pp. 2044–2051, 2014.

[12] Jim Woodhouse, "Plucked guitar transients: Comparison of measurements and synthesis," *Acta Acustica*, vol. 90, no. 5, pp. 945–965, 2004.

[13] Montserrat Pàmies-Vilà, Ibrahim Atakan Kubilay, Dmitri Kartofelev, Mirko Mustonen, Anatoli Stulov, and Vesa Välimäki, "High-speed linecamera measurements of a vibrating string," in *Proceeding of Baltic-Nordic Acoustic Meeting (BNAM), Tallinn, Estonia*, 2014, vol. 1.

[14] Arthur Paté, Jean-Loïc Le Carrou, and Benoît Fabre, "Predicting the decay time of solid body electric guitar tones," *The Journal of the Acoustical Society of America*, vol. 135, no. 5, pp. 3045–3055, 2014.