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MEASUREMENT AND ANALYSIS OF NON-LINEAR DISTORTION OF ELECTRO-ACOUSTIC AND AUDIO SYSTEMS

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Abstract

A novel technique for analysis of nonlinear distortion of electro-acoustic and audio systems is presented. The method is based on the combination of a *Sweep sine Wave signal Technique* (*SWT*) and a *Multiple Input Single Output* (*MISO*) nonlinear modeling. The signal driving a system under test is a logarithmic sine wave sweep. The response of the system under test is acquired and used in a convolution with swept sine wave "inverse filter" response to obtain a set of transfer functions. This set of transfer functions provides information about harmonic distortion. These transfer functions are further processed. Then the system under test can be modeled as a *MISO* nonlinear model, consisting of a parallel combination of nonlinear branches containing filters and memory-less power-law distortion functions. Such a model facilitates determination of a nonlinear response to the input signal. This method was used to analyze transfer functions of various components of electro-acoustic systems. It was applied to testing of recording and reproduction equipment for audio signals, including equipment for their further digital signal processing. During testing of multichannel systems, the method enables evaluation of not only individual channels, but also their mutual interaction. A good correlation with traditional methods of distortion measurement has been shown.

1. INTRODUCTION

During digital signal processing (*DSP*) of measured signal, such as of noise and vibration, audio and multimedia including their transfer, it is necessary to know their transfer properties, both individual and the whole system. We can assume that the nonlinear distortion happens mainly during system overload. Test method used should diagnose both this nonlinear distortion and eventual further faults taking place in system under test. While we analyses the transfer of multimedia signals that include compression of audio signals, such as *MPEG*, we are dealing with testing of the whole system, but not including the analysis of compression signals from the psychoacoustic point of view.

Currently, the preferred methods for measurement and analysis of electro-acoustic systems are the *MLS* signals method and sweep sine wave method. The *MLS* method is very robust, exhibiting excellent signal to noise ratio, however it cannot deal adequately with the analysis of nonlinear systems. From this point of view the better choice appears to be the *SWT* method, which enables to analyze the nonlinear distortion as well. This method was used for measurement of various systems, as described in [1]. In our contribution we first mention the basic principles of *SWT* method, and then we deal in detail with advanced method of nonlinear system analysis that is a combination of the *MISO* and *SWT* methods. The practical examples of this method are presented.

2. SWEEP SINE WAVE SIGNAL TECHNIQUE

The measurement and analysis of linear systems using *SWT* was thoroughly published [2,3]. We use this method for the analysis of individual components of the electro-acoustic chain, starting with signal pickup, recording and playback. Here, in view of its use in the next part of our contribution, we will provide its brief description. The linear system under test is driven by a logarithmically swept sine wave signal $x(t)$

$$x(t)=\sin\left\{2\pi f_1 T\left(\ln\frac{f_2}{f_1}\right)^{-1}\left[\left(\frac{f_2}{f_1}\right)^{\frac{t}{T}}-1\right]\right\}, \quad 0\le t\le T \tag{1}$$

where T is a sweeping time of the signal in frequency range $f_1 \div f_2$.

To eliminate transient behavior at the beginning and end of the test signal, we limit both ends by a time window $w(t)$. To determine the impulse response $h(t)$ of a system under test we use a so-called “inversion filter” $f_i(t)$ that can be derived from the driving signal

$$\left[x(t)w(t)\right]\otimes f_i(t)=\delta(t). \tag{2}$$

Using relations (2) we obtain the impulse response $h(t)$ of the linear system under test

$$h(t)=f_i(t)\otimes y(t). \tag{3}$$

From Eq. (3) it can be seen that the impulse response of the tested system $h(t)$ may be obtained as a convolution of the inversion filter $f_i(t)$ with the response of a system under test $y(t)$. If the system under test produces non-linear distortion, we can describe it as a non-linear memory less system. Such a system can be described by N^{th} order Volterra kernels [1]. The whole response $h(t)$ can be separated into partial impulse responses $h_i(t)$. The distorting components $H_i(\omega)$ in frequency domain are obtained from appropriate partial impulse responses using Fourier transform $FT\{h_i(t)\}$.

3. MULTIPLE INPUT SINGLE OUTPUT NON-LINEAR MODEL

3.1 Nonlinear Model

Many methods for identification of nonlinear systems have been developed, such as the Volterra Series [4], neural networks [5], NARMAX models [6], the MISO model [7] and others. All these techniques are comprised of analysis of the nonlinear system and identification of the nonlinear model, corresponding to the measured nonlinear system.

One of the best known nonlinear models is the Volterra Series model [4]. The Volterra theory states that any time-invariant, nonlinear system can be modeled as an infinite sum of multidimensional convolution integrals of increasing order. The convolution integrals can be represented by multidimensional kernels. The Volterra kernel is a function of several variables. Depending on the order, the model may contain a lot of coefficients, needed to describe the system.

The MISO model, used by Bendat [7], consists of a parallel combination of nonlinear branches containing linear filters and zero-memory nonlinear systems. If the zero-memory nonlinear systems of the MISO model are represented by the power-law distortion functions, the model corresponds to the Volterra subclass. Such a model is shown in Fig. 1. The output of the nonlinear system can be written as

$$y(t) = \sum_{n=1}^{N} \int_{0}^{\infty} x^n(t-\tau) g_n(\tau) d\tau, \tag{4}$$

where $x(t)$ is the input signal and $g_n(t)$, or $G_n(f)$ in frequency domain, represents the linear filter of the *n-th* branch.

According to Bendat [7], a nonlinear system based on a nonlinear equation can also be modeled in a *MISO* framework. In some cases, the nonlinear system may be modeled as a memory linear part following the nonlinear memory-less operation. This hypothesis is further considered.

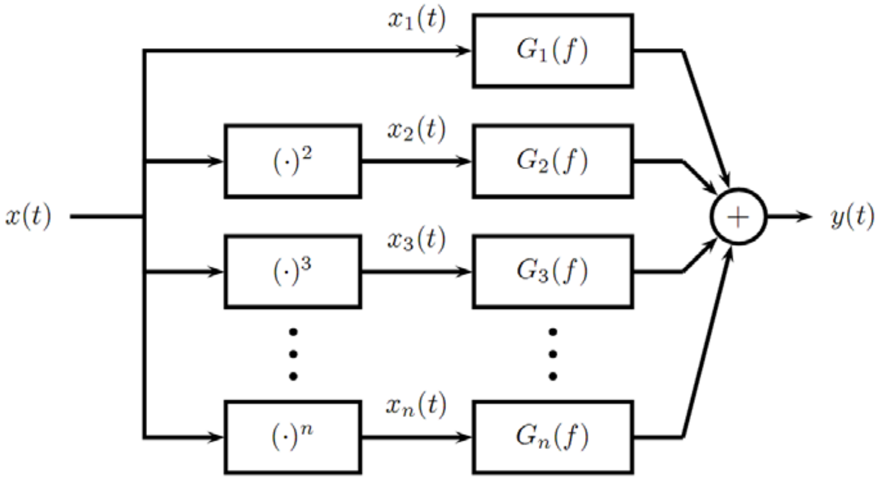

Figure 1. *MISO* nonlinear model. Each branch consists of power law function and a linear filter.

3. 2 Estimation of linear filters of the *MISO* model

The basic swept-sine measurement method can be modified to obtain the nonlinear *MISO* model and its linear filters $G_n(f)$. Using SWT technique described in section *2*, we can get the set of individual nonlinear distortion products $H_n(f)$. The distortion products $H_n(f)$ represent the coefficients of higher harmonic components and the linear filters $G_n(f)$ represent the coefficients of power series.

The relation between the harmonic components and their *n-th* powers must be calculated, using the trigonometric power formulas [8]

$$\sin^{2n+1} x = \frac{(-1)^n}{4^n} \sum_{k=0}^{n} (-1)^k \binom{2n+1}{k} \sin\left[(2n+1-2k)x\right], \tag{5}$$

$$\sin^{2n} x = \frac{1}{2^{2n}} \binom{2n}{n} + \frac{(-1)^n}{2^{2n-1}} \sum_{k=0}^{n-1} (-1)^k \binom{2n}{k} \sin\left[2(n-k)x + \frac{\pi}{2}\right]. \tag{6}$$

These equations describe the odd and even powers of the harmonic signal. Both equations can be rewritten to the matrix form

$$\begin{pmatrix} \sin x \\ \sin^2 x \\ \sin^3 x \\ \vdots \end{pmatrix} = \mathbf{A} \begin{pmatrix} \sin x \\ \sin 2x \\ \sin 3x \\ \vdots \end{pmatrix} + \mathbf{B}\ , \tag{7}$$

where the matrix ***A*** corresponds to the coefficients used for the transformation between the distortion products $H_n(f)$ and linear filters $G_n(f)$ from the *MISO* model. This transformation is shown in Eq. (8)

$$\begin{pmatrix} G_1(f) \\ G_2(f) \\ G_3(f) \\ \vdots \end{pmatrix} = \left(\mathbf{A}^T\right)^{-1} \begin{pmatrix} H_1(f) \\ H_2(f) \\ H_3(f) \\ \vdots \end{pmatrix} . \tag{8}$$

The matrix ***B*** corresponds to the first term of Eq. (6) and represents a mean value of the signal. This mean value is the product of even powers. This shows that even if the input signal has a zero mean value signal, the output signal of a nonlinear system does not need to have a zero mean. The linear filters $G_n(f)$ of the *MISO* model, pictured in Fig. 1, complete the nonlinear *MISO* model.

3.3 The *MISO* Model Verification

To verify the nonlinear *MISO* model of the nonlinear system under test, any kind of identical signal is input to both, the real nonlinear system and its *MISO* model, and then their output signals are compared. One of the criteria used for evaluation of nonlinear systems is the intermodulation distortion. It is based on applying an input signal consisting of two or more different frequencies. The output signal of the nonlinear system under test then consists not only of these fundamental frequencies, but also of the product of the intermodulation of these original fundamentals. The same signal, consisting of two different frequencies, is brought to the input of the nonlinear system and to the input of *MISO* model. The output signals, or its spectral representation, can be compared.

4. MEASUREMENT AND ANALYSIS OF ELECTRO-ACOUSTIC AND AUDIO SYSTEMS WITH NON-LINEARITIES

The combined method *SWT-MISO* was applied for analysis of electro-acoustic transducers and audio recording and playback equipment. As an example of usage for electro-acoustic transducers we show the analysis of performance of a midrange loudspeaker. A novel approach for frequency modulated transmission signals (*FM*) analysis and digital transmission of multimedia audio and video signals is then introduced. For measurement and analysis of electro-acoustic and audio systems with non-linearities we designed a custom program within the *MATLAB*® software. A computer generated test signals $x(t)$ is output through an external I/O measuring card. Responses of the devices under test are recorded on disk. Another software, e.g. *ADOBE AUDITION*®, can be used immediately in real time for observation of *DUT* performance, both in time and frequency domain. Further detailed analysis of recorded *DUT* responses $y(t)$ is then performed by *SWT-MISO* method.

4.1 Measurement and Analysis of Transfer Functions of Loadspeakers

To illustrate the application of combined *SWT-MISO* method we selected a measurement of nonlinear distortion of a midrange loudspeaker, used for power excitation of acoustic systems. This transducer is used over a frequency range of 50-4000 Hz with a maximum power of $100\,W$. The response of the *DUT* was picked up by a $1/4''$ microphone. The test was performed in a semi-anechoic chamber. We used a sweeping signal $0,5 \div 5\,kHz$ with a period of $T = 10\,\text{sec}$. and a sampling frequency $f_s = 96\,kHz$. Fig. 2 shows the measured impulse response $h(t)$, which can be divided into following partial components $h_i(t)$. The impulse response $h(t)$ is shown as absolute value $|h(t)|$ in *dB* scale, for a better graphic resolution. By application of a Fourier transformation $FT\{h_i(t)\}$ we obtain the frequency spectra $H_i(f)$ that are depicted in Fig. 3.

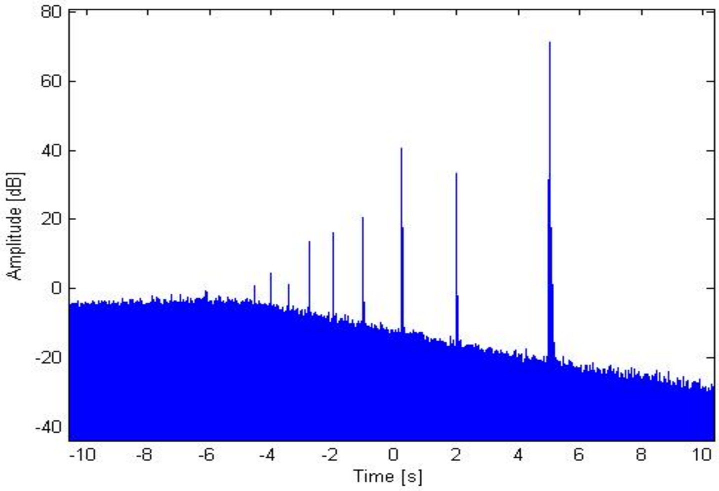

Figure 2. Partial impulse responses $|h_i(t)|$ of the loudspeaker under test.

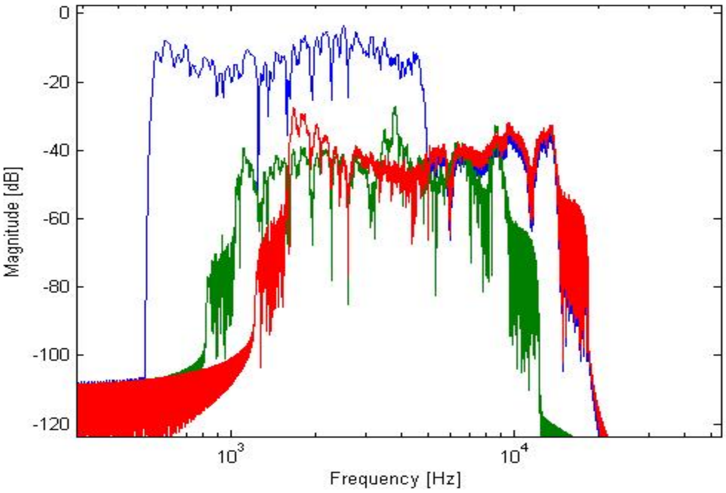

Figure 3. Partial transfer functions $|H_1(f)|$ (blue), $|H_2(f)|$ (green) and $|H_3(f)|$ (red) of the loudspeaker under test.

4.2 Measurement and Analysis of Equipment for Transmition of Audio and Multimedia signals

Nowadays, in addition to application in electro-acoustic transducers and recording and playback equipment, *DSP* is used also in terrestrial and satellite channels for transmission of audio and multimedia signals. It is for this reason that we are dealing with a possibility of applying *SWT-MISO* method for testing of the above mentioned transmission paths. As an example we show here the testing of *FM* transmission. The test method is similar to the previously described transducer measurement. The test signal was applied to the transmitter input, frequency modulated and subsequently evaluated by a test receiver. The samples of measured impulse response and the transfer function of the whole system are in Fig. 4 and 5.

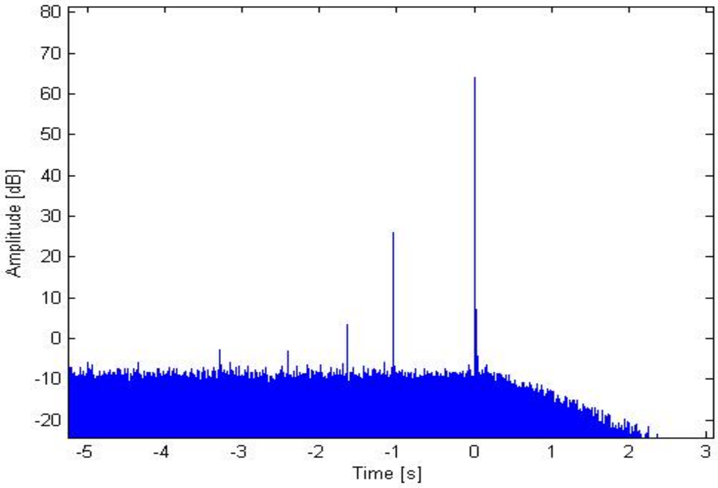

Figure 4. Illustration of partial impulse responses $h_i(t)$ of an *FM* transmission channel.

The samples of measurement and analysis of systems pictured in Fig. 2 to 5. give us the basic information about the combined *SWT-MISO* method. This method was successfully used for measurement of various electro-acoustic transducers and other audio equipment.

The more detailed description is beyond the scope of this contribution. The application of the method for measurement of transmission equipment as presented here is new. Our goal for the future is the development of a method for analysis of the total audio chain as a whole.

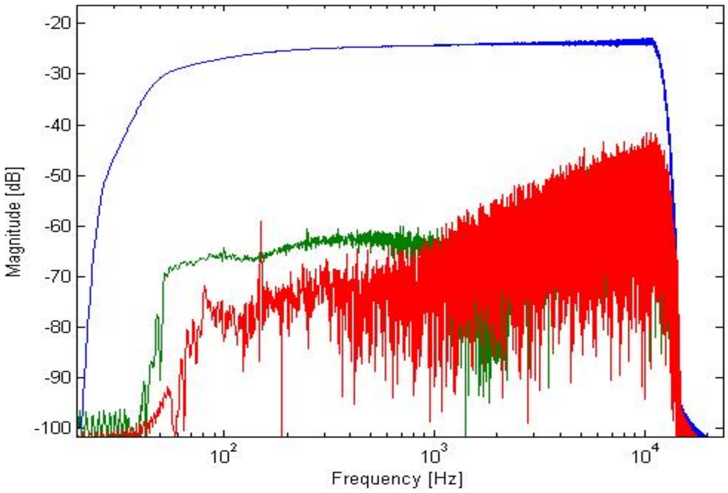

Figure 5. Illustration of partial transfer functions $|H_1(f)|$ (blue), $|H_2(f)|$ (green) and $|H_3(f)|$ (red) of an *FM* transmition channel.

5. CONCLUSIONS

The MISO method, modified by SWT technique, uses the same nonlinear model, but differs in principles of measurement. The excitation signal used for the original MISO method [7] is the white noise with zero mean value and defined standard deviation. The nonlinear powers of the input signal $x_1(t), x_2(t), ..., x_n(t)$ are calculated and form the set of inputs of the linear *MISO* system. The linear filters $G_n(f)$ of the linear part *MISO* model are estimated from power and cross-spectral densities.

The advantage of this method is that the *MISO* model, representing the measured nonlinear system, is able to reconstruct any input signal with amplitude less then three times the standard deviation of the excitation signal. The use of white noise as the excitation signal may cause some problems when an external noise appears. The external noise is must not be correlated with any of the branches of the *MISO* model, otherwise the estimation of the linear filters may fail. For that reason the sweep-sine method [1,2], originally used for nonlinear system analysis only is combined with the *MISO* modeling. The sweep-sine method is very robust method with a high accuracy even for low signal-to-noise ratios. Combining both methods and taking the advantages of both, the new method keeps its robustness and can yield a nonlinear model of the measured system. This model can be used further for simulations of the measured system, or conversely, to determine appropriate nonlinear filtration in order to minimize the nonlinear distortion.

The above described method was successfully used for measurement of transfer functions and analysis of electro-acoustic transducers and other components of audio systems. The results obtained from the model show a good agreement with results obtained by precision laboratory measurements on the real object. A new application is the use of this method in evaluation of a complete FM transmission chain as well as digital multimedia broadcast.

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