COMPARISION OF MEASUREMENTS METHOD FOR ANALYSIS ELECTRO-ACOUSTIC SYSTEMS WITH NON-LINEARITIES

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ABSTRACT
The great level of excitation of electro-acoustic systems may cause a nonlinear distortion of signal, which has an effect on its perception. For analysing and monitoring functionality of such a system there are two methods, Multiple-Input Single-Output method for nonlinear systems and a method utilizing sweep sine wave signals, which are compared in this contribution. The first method uses a stationary Gaussian random process as the input signal. Then a nonlinear system is represented by a multiple inputs linear system and additional nonlinear inputs are determined according to decorrelated power series expansion of the main input signal. The second method uses a logarithmic sweep signal as the input signal. The impulse response of the system is obtained by a convolution of the system response to the input sweep signal and the "inverse filter". The impulse response consists of several distorted partial components. These two methods have been compared from the accuracy of the algorithmic point of view and also by measuring by an electro-acoustic system.

INTRODUCTION

One of the goals of today's research in the field of sound systems is to identify its nonlinearities and model them, for better comprehension of their principles. In this project, two different methods have been tested. The first method called Multiple Input – Single Output method (MISO) is better known in the field of mechanical engineering, where it is applied using the Reverse-Path MISO, assuming some knowledge of nonlinear systems. The Direct-Path MISO method used in this work has been adopted to be able to identify nonlinear systems without any knowledge of the nonlinear system. The result of this method is a nonlinear model based on power series. The second method, called Sweep-Sine technique, is well known in the field of acoustics and electro-acoustics. It is based on swept sine excitation of the system under test and subjecting the response to a convolution process with an inverse filter. The result of this method is a sequence of impulse responses, which can be expressed as partial frequency responses with higher harmonic components [1], [2]. The goal of this work is to compare these two methods.

For modelling of an output signal of a nonlinear system, we can either introduce a harmonic distortion by adding distorted components as higher harmonics themselves, or by modelling a nonlinear system by nonlinear block parts. The second method is more common however the anti-aliasing rule must be respected. If an output signal of a nonlinear system is modelled, the absolutely highest frequency of the input signal must not exceed the frequency $f_s/(2m)$, where f_s is the sampling frequency and m is order of power series of the nonlinear input-output characteristics. As a power series model is used in one of the tested methods and a method of adding distorted components is used in the other one, it is appropriate to define a relation between n-th power of the fundamental sine signal and its n-th harmonic. Since only the odd-nonlinearity will be tested, only odd-power components are expressed here

$$\sin^{2n-1}(\alpha) = 2^{2-2n} \sum_{k=0}^{n-1} (-1)^{n-k-1} \binom{2n-1}{k} \sin[(2n-2k-1)\alpha] \qquad \text{(Eq. 1)}$$

DESCRIPTION OF MEASURING METHODS

Multiple Input – Single Output Method
The Direct Path MISO method used in this work is based on a blind identification of nonlinear system. The nonlinear MISO model (see Figure 1) is represented by an equivalent Multiple-Input Single-Output linear model, where the inputs are nonlinear contributions of the original input signal. The model consists of branches, where the first branch represents a linear part and another branches are nonlinear contributions with memory part expressed by a linear filter [3]. In this work, zero-memory nonlinear parts of branches are chosen according to power series, but generally it can any kind of zero-memory nonlinearity. The input signal $x(t)$ is in a form of stationary Gaussian random process. General problem of solving MISO linear system is a correlation between inputs of the system [4]. The same problem arises here, but having a white noise as the input signal, the decorrelation can be mathematically expressed [3].
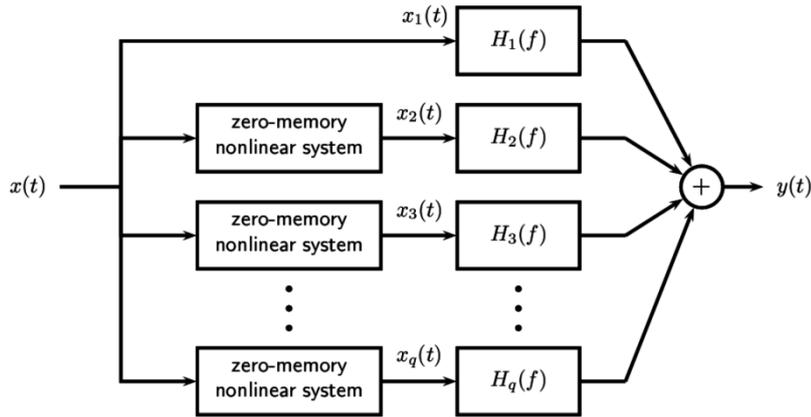

Figure 1.- Multiple Input Single Output nonlinear model

Swept-Sine Technique
The swept signal method is based on relation between of the impulse response $h(t)$ of the system under test and its response $y(t)$ on the system output

$$h(t) = f_i(t) \otimes y(t), \qquad \text{(Eq. 2)}$$

where a so-called "inversion filter" $f_i(t)$ can be derived from the driving signal $x(t)$

$$x(t) \otimes f_i(t) = \delta(t). \qquad \text{(Eq. 3)}$$

Log-swept driving signals can be written [1], [5]

$$x(t) = \sin\left\{2\pi f_1 T \left(\ln\frac{f_2}{f_1}\right)^{-1} \left[\left(\frac{f_2}{f_1}\right)^{\frac{t}{T}} - 1\right]\right\}, \ \ 0 \leq t \leq T. \qquad \text{(Eq. 4)}$$

Electro-acoustic systems under large signal excitation will produce non-linear components. Such a systems can be described by order Volterra kernels [1], [6]. By an appropriate selection of swept signal method parameters we can make this method suitable for analysis of systems with non-linearities as well. In that case, the distorting components can be separated into partial impulse responses of the system.

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COMPARISON OF NUMERICAL ACCURANCY OF BOTH METHODS

To test both methods from numerical accuracy point of view we used a nonlinear system with a defined distortion. As both methods use different kind of input signal and method of processing, we have decided to define a tested nonlinear system as a system whose response to a sine signal with normalized amplitude is the same input signal with following harmonic components: third harmonic 3%, fifth harmonic 1% and seventh harmonic 0.5%. This definition of harmonic distortion can be expressed in a form of single-row matrix

$$[\mathbf{K}] = [1 \quad 0.03 \quad 0.01 \quad 0.005].$$
(Eq. 5)

For testing we consider the highest distorting component the 7th harmonic of the fundamental frequency. The numerical simulation has been also limited to odd harmonics frequency. Whereas a Sweep-Sine method can be tested on a distorted signal made by adding higher distorted components, the MISO method has to be tested on a model with multiple inputs and outputs, so the following modification is required. Assuming this limitation to the 7th harmonic, the equation (1) can be rewritten in matrix form

$$\begin{bmatrix} \sin(\alpha) \\ \sin^3(\alpha) \\ \sin^5(\alpha) \\ \sin^7(\alpha) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \dfrac{3}{4} & -\dfrac{1}{4} & 0 & 0 \\ \dfrac{5}{8} & -\dfrac{5}{16} & \dfrac{1}{16} & 0 \\ \dfrac{35}{64} & -\dfrac{21}{64} & \dfrac{7}{64} & -\dfrac{1}{64} \end{bmatrix} \cdot \begin{bmatrix} \sin(\alpha) \\ \sin(3\alpha) \\ \sin(5\alpha) \\ \sin(7\alpha) \end{bmatrix},$$
(Eq. 6)

that can be simplified into the following form

$$[\mathbf{P}] = [\mathbf{A}] \cdot [\mathbf{S}].$$
(Eq. 7)

Matrix **A** expresses a relation between higher harmonic components $\sin(n\alpha)$ and power series components $\sin^n(\alpha)$. As we have defined a nonlinear system by a single-row matrix **K**, power series coefficients **D** can be obtained as

$$[\mathbf{D}] = [\mathbf{K}] \cdot [\mathbf{A}]^{-1},$$

$$[\mathbf{D}] = [1.175 \quad -0.6 \quad 0.72 \quad -0.32].$$
(Eq. 8)

Now a power series model for testing a MISO method is defined by matrix **D**. The following equation defines the output of the tested nonlinear system

$$y = 1.175x - 0.6x^3 + 0.72x^5 - 0.32x^7.$$
(Eq. 9)

For MISO method a white noise input signal with frequency range of 2 kHz has been used. Sampling frequency has been chosen $f_s = 44.1\,kHz$ according to CD format. The distorted coefficients for testing a Sweep-Sine method are defined by matrix **K**. For the test of the method from numerical accuracy point of view we used swept signal in the $20\,Hz - 2\,kHz$ range with sweeping time $T = 20\,sec$ and sampling frequency $f_s = 44.1\,kHz$. The sufficient length of input signal has been chosen in order to make possible the separation of a total impulse response (IR) $h_d(t)$ consisting of partial components

$$h_d(t) = h_1(t) + h_3(t) + h_5(t) + h_7(t).$$
(Eq. 10)

Multiple Input – Single Output Method
The coefficients of matrix **D**, estimated using the MISO method, are pictured in the Figure 2a. As the method has been tested on numerically exact system defined by equation (9) and the order of power series has been set up according to maximum order of tested system, the results are straight lines with values of matrix **D**. Figure 2b shows the output signal spectrum of a MISO model obtained from the identification, where the input was a sine.
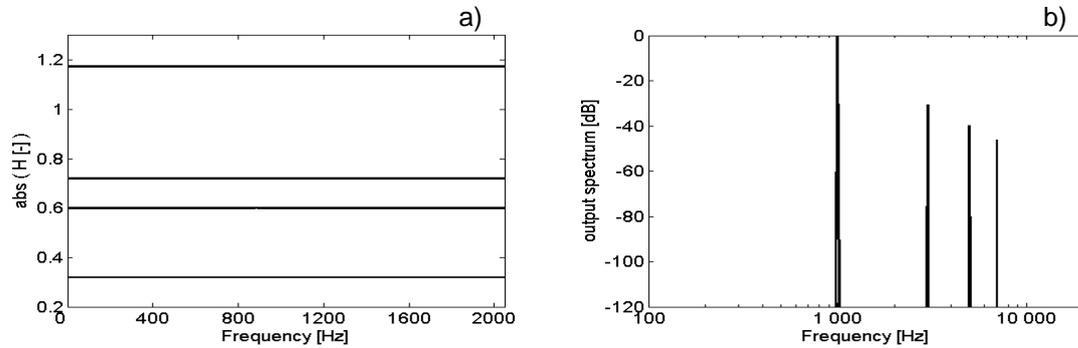

Figure 2. - a) The coefficients of matrix **D**, estimated using the MISO method;
b) Frequency analysis of response to a $1\ kHz$ sine wave

Swept-Sine Technique
Results of the numerical analysis are shown in Figure 3. As a linear-view interpretation of IR is not sufficiently accurate for a detail analysis (see Figure 3a) the total IR is plotted in logarithmic scale $20 \cdot \log_{10}|h(t)|$ (Fig.3b). Using Fourier transformation $FT\{h(t)\}$ we get the frequency spectra $H_{h1}(\omega)$, $H_{h3}(\omega)$, $H_{h5}(\omega)$ and $H_{h7}(\omega)$ that are pictured in Figure 3c. In Figure 3d we can see the frequency analysis of response to the sweep signal at a specific time moment $(f = 1\ kHz)$. Observing Figure 3, we can get an idea about the accuracy of the measuring method, which is higher than 0.1 dB.
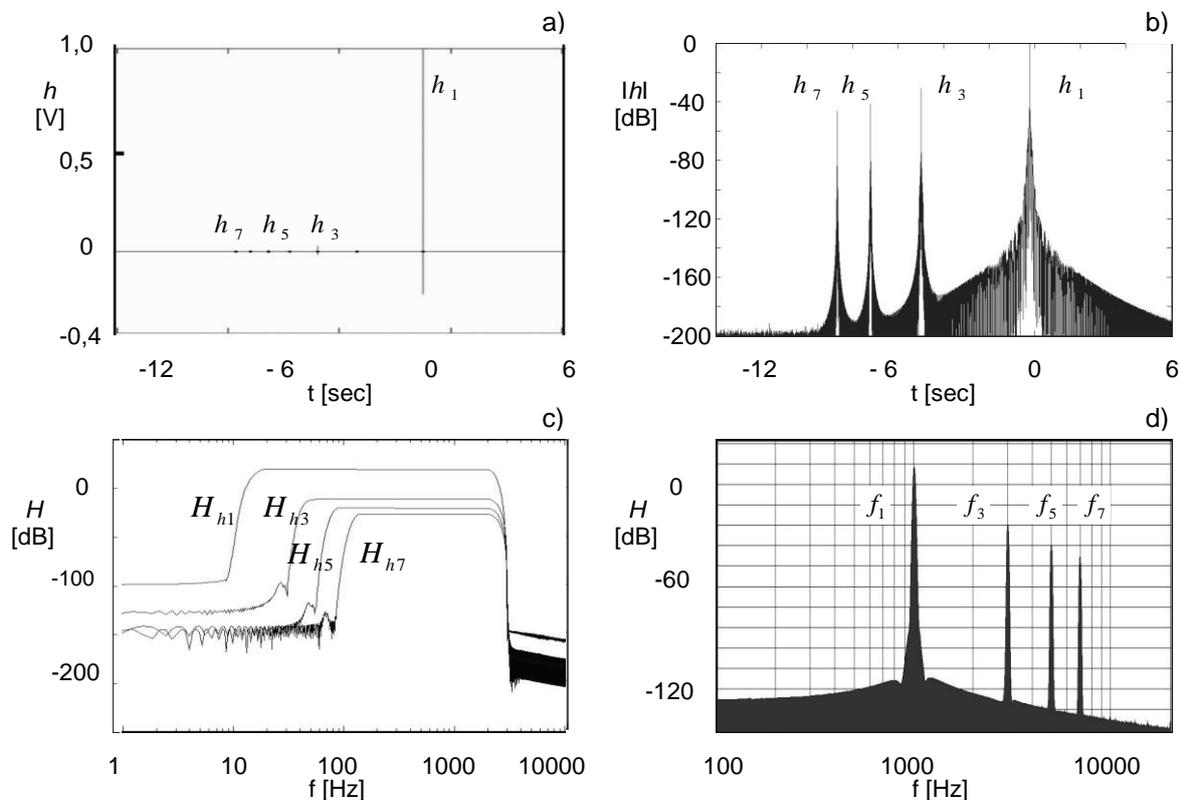

Figure 3. - a), b) Impulse responses $h_i(t)$ obtained by distorted test signal (linear and log scale); c) Transfer functions $H_{hi}(\omega)$ obtained from distorted test signal; d) Frequency analysis of distorted signal in the region of fundamental frequency $1\ kHz$

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IDENTIFICATION OF AN AUDIO LIMITER

For verification of functionality and accuracy of both methods, an Audio Limiter (AL) has been tested. Input signals were generated and recorded on a CD. Signals were then applied to the tested system input. The output signal has been recorded using a measuring card in PC workstation and then processed. The block diagram of the test system is shown in Fig. 6.
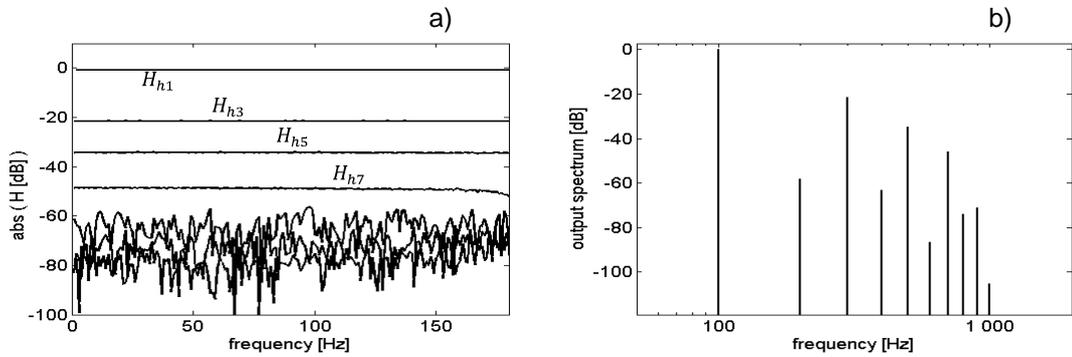
Multiple Input – Single Output Method

Figure 4. – a) Frequency responses of distorted harmonic components;
b) Frequency analysis of system response to a $100\ Hz$ sine wave
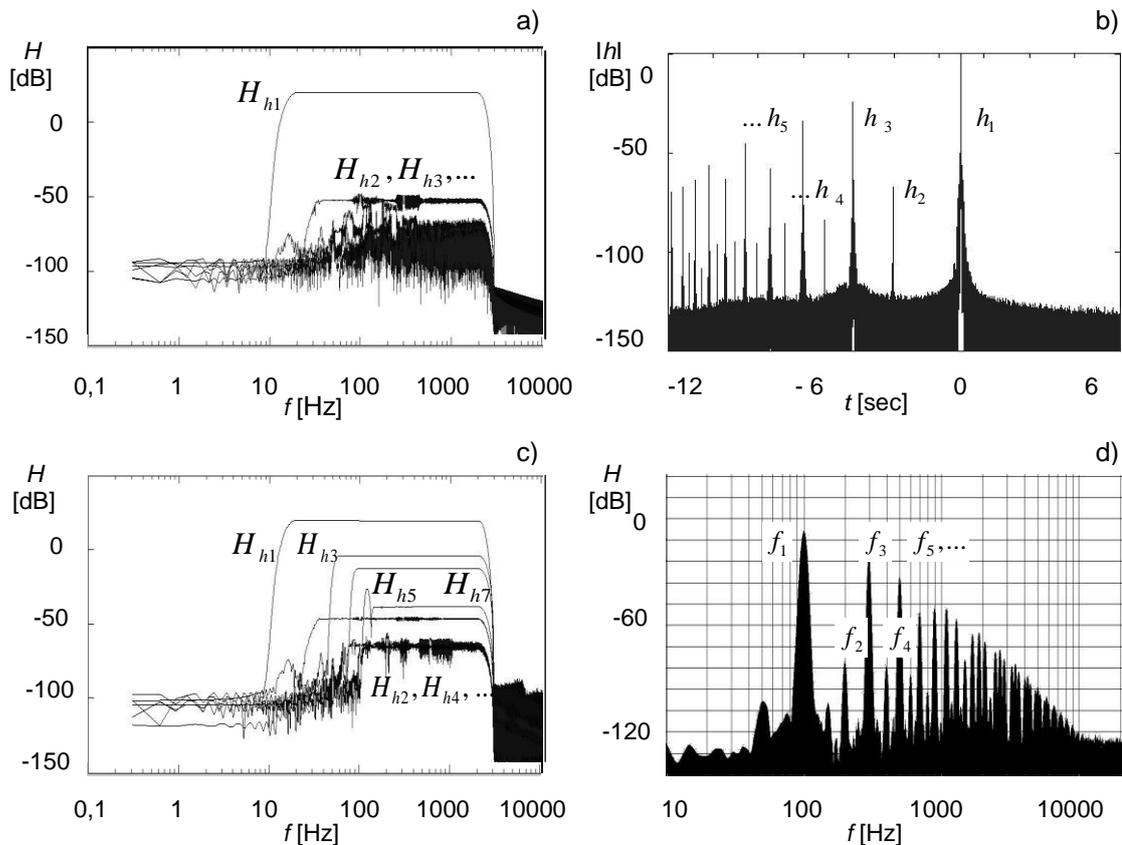
Swept-Sine Technique

Fig. 5. – a) Frequency responses of non-overload system $H_{h1}(\omega)$ and the 2nd, 3rd,.. harmonic components; b) Partial impulse responses $h_i(t)$ of overload system; c) Frequency responses $H_{hi}(\omega)$ of overload system; d) Frequency analysis of overload system in the region of fundamental frequency $100\ Hz$

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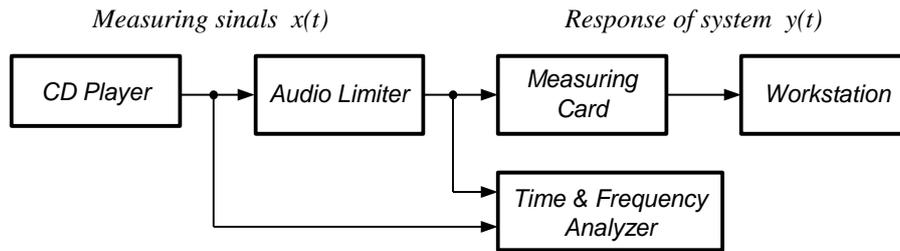*Measuring sinals x(t)*   *Response of system y(t)*

Figure 6. - Block diagram of measurement and analysis of the audio limiter under test

Figures 4 and 5 show the results obtained from testing of Audio Limiter using MISO (Figure 4) and Sweep-sine method (Figure 5). It is obvious from results of both methods that only the odd harmonic components are perceptible, whilst the even components are more or less hidden in the noise-level. In Figure 5a a results of non-overloaded audio limiter is shown.

CONCLUSIONS
Two methods for analysis of nonlinear systems, MISO and Sweep-Sine method have been tested by numerical simulation and on a real audio system. The MISO method shows better results in a numerical testing, where the model obtained by numerical identification has been exactly the same as the nonlinear system under test. That depends on choice of the tested system. In this particular case only polynomial nonlinearity of order 7 has been used. The Sweep-sine method had also a perfect numerical accuracy (better then 0.1 dB). The Sine-sweep method has shown its robustness in testing of real audio systems. Using this method, one can obtain all the nonlinear components (including higher harmonics). The MISO method has used only 10 order power series, so only 10 first harmonics can be seen. An inaccuracy of this method may be also affected be the presence of noise in tested system that might be correlated with tested signal. Both methods have identified the nonlinear system correctly and were able to identify nonlinearities in the whole frequency band.

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