Abstract

This paper deals with the properties of loudspeaker with direct digital to analog conversion, the basic goal being to replace D/A converter, amplifier, and classic speakers. The advantages of this system should be lower operating costs and greater applicability. The main part of this paper is aimed at analyzing the signals acquired from the theoretical model of the digital loudspeaker arrays (DLA) with regards to the arrangement of elements on the transducer field and the point of observation.

1 Introduction

If we consider an electro-acoustic chain as a process of recording, signal processing and reproduction of an acoustic signal. The last components in this chain, which are still analog, are microphone, amplifier and loudspeaker (respectively headphones). Nowadays recording and signal processing are digitized and it is discussed that digitalization of all systems could be achieved in the process of time.

Substitute for an amplifier and loudspeaker should be a digital loudspeaker with digital input signal. A direct analog-digital conversion is then achieved in a fluid (gas) thanks to superposition of acoustics waves, next to other physical phenomena. The first person who dealt with digital loudspeaker was J. L. Flanagan [1] in 1980.

2 Digital Loudspeaker Arrays

The main idea of Digital Loudspeaker Arrays is to transmit the parallel digital stream using any number of elements. The aim is to obtain analog acoustic signal after superposition of acoustics waves. The basic principle is to use same transducers (elements) of the loudspeaker array. Each bit corresponds to \(2^{n-1}\) elements, where \(n\) is the bit significance. Due to distances \(d\) between elements, the distances \(r_i\) from the point of observation \(P\) to all of elements are not same and signals are superposed with a different time delay.

![Figure 1: Time delays due to different distances](image)

The basic idea of digital loudspeaker arrays (DLA) is to divide a digital PCM signal to single bit signals of a parallel stream and bring them to the input of digital loudspeaker arrays. The least significant bit or its signal is brought to one element, the second least significant bit signal to two elements, the third to four and the \(n\)-th bit signal to \(2^{n-1}\) elements (alternatively to their multiples). Overall we need \(1 + 2 + 4 + ... + 2^{n-1} = 2^n - 1\) elements.

It is better for realization of digital loudspeaker to use a three-level digital signal with levels \((+U, 0, -U)\). This is
why the most significant bit, or it’s bit signal, is used as a sign bit. Then the number of the required bit signal is decreased by one and the number of required elements will be $2^{n-1} - 1$. The question is how to arrange this number of elements in a plane or in space.

### 3 Arrangement of elements

The most suitable arrangement of elements (it a plane) is a square-grid by technological reasons. However any other arrangement such as a circular one can be considered. The optimal arrangement of elements in the square-grid seems to be the arrangement where corners of the square-grid are unoccupied and the array of elements forms a circular shape. In this paper the arrangements of elements according to paper [4], in which five different arrangements were tested. Four of them are regular (the elements were placed into the square-grid according to a rule) and one of them was random (the elements were placed into the square-grid randomly). In this paper the next random arrangement was tested. The elements are selected randomly at each sample interval. This arrangement can be called randomized digital array or array with space diversity.

![Figure 2: Example of a regular arrangement](image)

### 4 Consequence of different distances

Non-linearities arise as a result of signal decomposition into parallel bit-stream signals, their time delay and following superposition. Figure 3 shows the sine input signal before digitalisation (top), and the same signal after digitalisation and passing through a digital loudspeaker array (bottom).

![Figure 3: Example of input and output signals (time domain)](image)

Figure 4: Example of input and output signals (frequency domain)

For this example a 10-bit digital loudspeaker has been used. One of the regular arrangements has been used, the distance between elements was 1cm and the point of observation was chosen in the axis of the array at a distance 10cm.
5 Choice of the kernel

Take into consideration a three-level bit-signal with levels $U_0, +U$ as shown in the next figure.

![Figure 5: three-level bit signal](image)

Simplification for mathematic formulation can be done. The signal $b(t)$ should be expressed by convolution of sequence of scaled Dirac pulses $a(t)$ at each sample interval $n \cdot T_s$ and rectangle signal $g(t)$ with duration $T_s$.

\begin{equation}
    b(t) = a(t) * g(t).
\end{equation}

Generally we can choose any shape of kernel $g(t)$. The principle of a digital loudspeaker is using the digital bit-signal with two or three level signal is used. The rectangular shape of kernel $g(t)$ with duration $T_s$ will be considered, even if it is not efficient in the frequency domain. Even though it is suitable to separate the bit-signal $b(t)$ into a sequence of scaled Dirac pulses $a(t)$ and kernel $g(t)$. The next figure shows the flow diagram of such a system, where $h(t)$ represents the impulse response of a single element of a digital loudspeaker array and the block „fluid” represents the block of superposition of mutual delayed bit signals.

![Figure 6: Flow diagram of a digital loudspeaker](image)

The input signal of the block fluid can be expressed by convolution of single blocks

\begin{equation}
    y_m(t) = h(t) * g(t) * a(t),
\end{equation}

from which can be derived

\begin{equation}
    y_m(t) = H(t) * a(t),
\end{equation}

where $H(t)$ is the response of elements $h(t)$ including the kernel $g(t)$. The following derivation shows the advantage of using a kernel

\begin{align}
    s(t) &= \sum_m \int_{-\infty}^{\infty} H(\tau) \cdot a_m (t - \tau - \Delta m) \, d\tau. \quad (4) \\
    &= \sum_m \int_{-\infty}^{\infty} H(\tau) \cdot a_m (t - \tau - \Delta m) \, d\tau. \quad (5) \\
    &= \int_{-\infty}^{\infty} H(\tau) \sum_m a_m (t - \tau - \Delta m) \, d\tau. \quad (6)
\end{align}

The result can be expressed by convolution

\begin{equation}
    s(t) = H(t) \ast \sum_m a_m (t - \Delta m). \quad (7)
\end{equation}

This expression means that the impulse response of elements and the shape of the kernel $H(t)$ can be applied after the superposition of the sequences of scaled Dirac pulses.

Diagram 6 can be redrawn with using equation (7) into the next figure. The diagram has no logical coherence; the intermediate results between the blocks have no physical meaning; the output signal $s(t)$ is mathematically the same as the output of the flow diagram (above).

![Figure 7: Flow diagram after modification](image)

There is a noticeable consequence of choosing the kernel and impulse response of the elements, which is the result of equation (7). By choosing the function $g(t)$ and $h(t)$, or rather $H(t)$, it is possible to remove only the components above frequency $f_s/2$ (where $f_s$ is sampling frequency). For perfect reconstruction and filtration of these components, it is necessary to choose an impulse response $H(t)$, which has a constant (not-zero) transfer function up to frequency $f_s/2$ and a zero transfer function above this frequency. This demand can be satisfied only by using the function

\begin{equation}
    H(t) = \frac{\pi}{T_s} \sin\left(\frac{2\pi f}{T_s}\right).
\end{equation}
By using this function the undesirable components over the frequency $f_s/2$ can be removed. To keep the basic idea of a digital loudspeaker, the shape of kernel should be chosen as a rectangular signal and the possibility to change the response $H(t)$ is able only by changing the impulse response of the elements $h(t)$. The kernel $g(t)$ with any kind of shape can be chosen; then it is not a digital, but a hybrid system. This is the same as adding another block of filtration between block of kernel and the array of elements.

However the problem with the components above frequency $f_s/2$ is not the major problem in the construction of digital loudspeaker arrays. The most undesirable components arise from non-linear distortion, which is impossible to remove by linear filtration or by choosing the shape of kernel $g(t)$.

6 Distortion

We can characterise a distortion by comparison of the harmonic input signal with the response to this distorted harmonic signal in the output of the system. The distortion is then defined by the effective values

$$ k = \frac{I_{\text{res}}}{I_{\text{out}}}, \quad (9) $$

where $I_{\text{out}}$ is the effective value of input signal and $I_{\text{res}}$ is the effective value of the "residual curve", which is given by removing the first harmonic (useful signal) from the output. The equation for the distortion can be rewritten into

$$ k = \sqrt{\frac{\sum_{n=0}^{N-1} |S_{n,\text{res}}|^2}{\sum_{n=0}^{N-1} |S_{n,\text{out}}|^2}}. \quad (10) $$

The dependency of the distortion on the frequency will change with the arrangement of elements, their distance and with the point of observation.

Figure 8: The dependency of the distortion on the frequency

The distortion is also angle-dependent.

Figure 9: The dependency of the distortion on the angle

When the angle is near to $0^\circ$, or in the axis of the loudspeaker array, the distortion is the least, but when the angle is further of the axis, the distortion increases. The increase of the distortion is given by the useful signal, which has a big decrease at high frequencies - the digital loudspeaker arrays appear to be a low-pass filter for the useful signal.

We can draw the dependency of the distortion on the frequency and angle into the one figure, by using the dependency on two variables $k(f, \varphi)$ and drawing it into the 3D figure. There is a noticeable difference between using the regular and the randomised arrangement. The area of higher frequencies and angles is very similar and it is not possible to use the digital loudspeaker in this area, due to high distortion. The distortion at the area of low frequencies is far lower when the randomised arrangement is used.
7 Results

Digital loudspeaker arrays seem to be a non-linear system with many undesirable components. It can be expressed by distortion, which rapidly increases with higher frequencies or angles out of the axis of loudspeaker arrays. This is caused by a big decrease of amplitude of the usefull signal, because the arrays of elements appear to be a lowpass filter for the usefull signal, whose cut-off frequency depends on point of observation.

In regard to distortion, the best arrangement is the randomised array where the elements are selected randomly at each sample interval.

Reference


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