Can nonlinear convolution improve damage characterization using acoustic methods?

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RÉSUMÉ. Ce travail a pour but de caractériser l'endommagement de matériaux homogènes et hétérogènes en utilisant une démarche expérimentale essentiellement basée sur une technique de traitement du signal. La méthode dite de « convolution non linéaire » est utilisée afin d'améliorer les mesures expérimentales permettant ainsi d'avoir accès simultanément à la composante linéaire du spectre de vibration, habituellement effectuées à l'aide d'un analyseur de réseaux, et les composantes harmoniques, inaccessibles via les moyens expérimentaux classiques. La méthode a été validée en suivant l'endommagement progressif d'un composite base polymère chargé de fibres de verre.

MOTS CLES: convolution non linéaire, élasticité non linéaire, caractérisation de l'endommagement

ABSTRACT. This work presents an original damage characterisation method of homogeneous and heterogeneous solids using a signal processing-based experimental approach. Nonlinear convolution method is used in order to improve experimental observations allowing to measure simultaneously the well known vibration spectrum, classically found using analysers, and harmonic spectrums which are out of reach when using the same analysers. The experimental approach has been validated to characterise a progressive damage corresponding to a glass fibre polymer-based composite

KEYWORDS: nonlinear convolution, nonlinear elasticity, damage characterisation

1. Introduction

Very often, nonlinear acoustic techniques revealed their high sensitivity to micro-structure evolutions of several kinds of materials at the time when no changes are observed on the usual linear acoustic parameters, namely attenuation and velocity. This is mainly due to the fact that elastic parameters corresponding to these materials (rocks, polymer-based composites, concrete, bone, etc.) are micro-strain amplitude dependent. This induced strain dependence is generally modelled by developing the elastic modulus K as:

$$K = K_0\left(1 + \beta\,\varepsilon + \delta\,\varepsilon^2 + \ldots\right) - \alpha\left(\varepsilon, \dot{\varepsilon}\right) \qquad (1)$$

where, K_0 is the linear modulus, ε is the strain, β and δ represent the classical quadratic and cubic nonlinear parameters, respectively which can be developed as a combination of 2nd, 3rd and 4th order elastic constants and α is the parameter of hysteretic nonlinearity.

From the acoustic wave propagation point of view, equation 1 formulates the nonlinear modulus in such a way that classical as well as hysteretic nonlinear behaviours are clearly differentiated. Indeed, when the micro-cracked material is excited with an acoustic perturbation of frequency f and amplitude ε, it generates higher frequency components $2f$, $3f$, etc. whose amplitudes are proportional to ε^2, ε^3, etc. However, when the micro-cracked material is nonlinear hysteretic, amplitudes of odd and even harmonics are seriously disturbed when compared to the classical nonlinear case (for instance, some of odd harmonics amplitudes are higher than even harmonics, and the third harmonic is proportional to ε^2, etc.) (Abeele *et al.* 2001, Gusev *et al.* 1998, Bentahar *et al.* 2006, Johnson *et al.* 2005).

The nonlinear behaviour of micro-cracked materials could be acoustically characterized by using either standing waves (resonance modes to determine α) or single frequency tones (harmonics generation to determine β and δ). As these two measurements cannot be performed simultaneously, in terms of a well controlled frequency excitation, we propose in the following section a signal processing method to make the simultaneous characterization possible.

2. Nonlinear convolution method

The input signal used for the analysis is an exponential swept sine signal, i.e. a signal exhibiting an instantaneous frequency $f_i(t)$ which increases exponentially with time (Fig. 1). Such a signal is also called an exponential chirp and is defined as:

$$x_s = \sin\left\{2\pi f_1 L\left[\exp\left(\frac{t}{L}\right)-1\right]\right\} \tag{2}$$

where, L is defined as

$$L = \frac{1}{f_1} Round\left(\frac{\hat{T} f_1}{\ln\left(\frac{f_2}{f_1}\right)}\right) \tag{3}$$

The properties of the swept sine signal are defined by start and end frequencies f_1 , f_2 and by the approximate time support $\hat{T}$ (Novak *et al.* 2009).

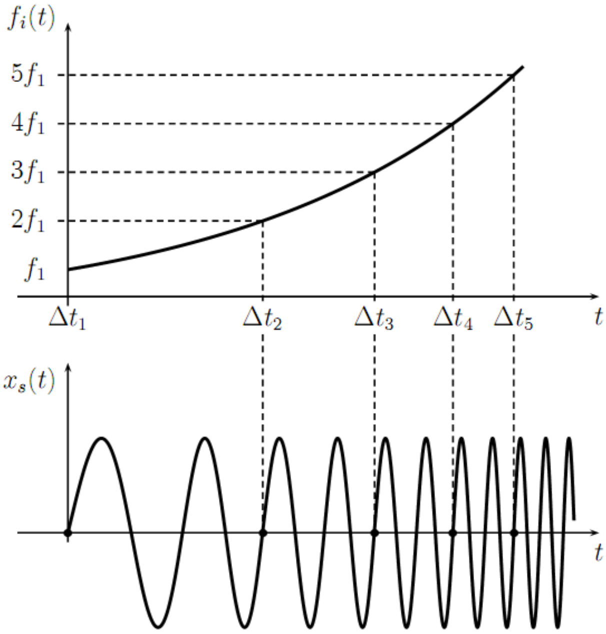

Figure 1. *Swept-sine signal $x_s(t)$ in time domain (below) with the time length chosen according to instantaneous frequency $f_i(t)$ (above).*

The basis of the method of identification is nonlinear convolution (Farina 2001). First, an inverse filter is generated as a time inverted input signal with decreasing amplitude (Farina 2001). Then, the ordinary linear convolution between the captured output signal and the inverse filter is calculated. The result of the convolution is a set of time shifted impulse responses called higher-order nonlinear impulse responses (Fig. 2). As the impulse responses are separated in time, they can be easily windowed. Afterwards, the Fourier Transform of each separated higher order impulse response can be calculated. The results are called higher order frequency responses $H_i(f)$. The i-th response corresponds to the frequency evolution in amplitude and phase of the i-th higher harmonic when exciting the system with a harmonic signal. Thus, the method can within one measurement of time length $\hat{T}$ characterise the nonlinear system in amplitude and phase not only for the fundamental harmonic as usual, but also for higher nonlinear harmonics. The principle of the method has been studied in detail in (Novak *et al.* 2009).

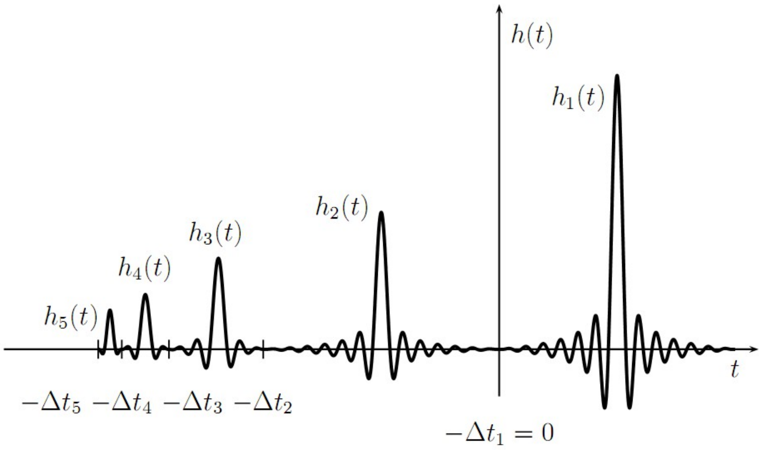

Figure 2. *Result of the nonlinear convolution process in the form of set of higher-order nonlinear impulse responses* $h_i(t)$.

3. Experimental Results

Figure 3-a shows the experimental set up. A cross ply composite beam is excited around its three first flexural modes using a low frequency shaker. The beam response is detected by an accelerometer placed close to its free end. The nonlinear convolution method provides additional information compared to the classical resonance method. Indeed, in the case of the dispersive flexural modes for instance, it is always interesting to have the behaviour of the harmonics corresponding to the different resonance modes and define new damage sensitive parameters. In that sense, figure 3-b shows the fundamental resonance response as well as its second and third harmonics.

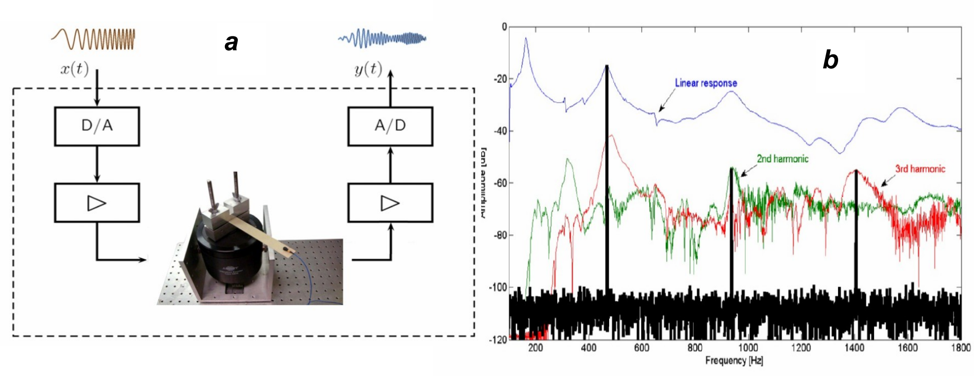

Figure 3. (a) Experimental device used for the nonlinear characterization; (b) Resonance response of the composite sample: the nonlinear convolution method allows to obtain the spectrums of the 2nd as well as the 3rd harmonics.

At a given excitation level, the existence of a second harmonic, that was absent at the intact state, is a damage indicator. This proves the existence of an interesting opportunity to follow the evolution of the 2nd harmonic amplitude as a function of the fundamental's one. On the other hand, we have found that the fundamental bending frequency (~180Hz) as well as its second harmonic (~360Hz), do not have the same evolution as a function of the excitation amplitude when damage increases. Indeed, Table 1 shows the relative frequency shift $\Delta f / f_0$ of three increasing damage states corresponding to a composite plate using step-off levels from 0 to 3 mm in 1 mm increment, where f_0 is the lowest amplitude resonance frequency and $\Delta f = f_0 - f$ where f is the resonance frequency at highest drive levels. As the increase of amplitude induces a softening in the composite properties we have $f_0 - f > 0$.

State	0	1	2	3
$\Delta f / f_0$ (%) (fundamental)	0.5	0.5	1.1	1.3
$\Delta f / f_0$ (%) (2nd harmonic)	0.5	0.5	1.5	3.7

Table 1. Relative frequency changes corresponding to the the fundamental flexural mode (~180Hz) and its second harmonic (~360Hz): $\Delta f = f_0 - f$ is the difference between resonance frequencies corresponding to lowest and highest excitation levels, respectively.

Beyond their sensitivity to damage creation and evolution (and hence their structural health monitoring potential), these very first results based on the nonlinear convolution method reveal to be an interesting tool for the study of the nonlinear behaviour of materials (hysteretic or classic). In that sense, we are developing this study on other materials, such as glass bars and metal based-composites, in order to understand in a better way the influence of an increasing damage on their nonlinear vibrations as well as the physics that lies behind them.

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