Identification of Nonlinear Systems: MISO modeling

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Abstract. *In this paper, the fundamentals of analysis of nonlinear systems using the direct path MISO method (Multiple Input / Single Output) with the input signal as a record from stationary gaussian (ergodic) random process with zero mean value will be introduced. The MISO technique for linear system is well known in many fields of science and technical applications and the first publications using MISO technique for nonlinear systems was presented by J.S.Bendat [1]. This technique has been used in some acoustical applications (for example in vibration analysis [3]).*
This paper will discuss the application of MISO method for general nonlinear system using a polynomial series. An easy experiment will be conducted to show the functionality of this method.

Keywords

Nonlinear, MISO, systems, identification, analysis.

1. Introduction

If the system to be estimated is not linear, the classical relationships used to identify linear systems are not verified and the theory becomes more complicated than in the case of linear systems. If the system is weakly nonlinear (which means that the nonlinear part of a system is very small, compared with the linear one) then the linear relations can be used for getting the approximation of Frequency Response Function, impulse response etc. In many areas of technical applications, nonlinear systems are often linearized for an easier computation. In some cases the nonlinear part of the system is too high and such an approximation may not be used. In such cases, another way of modeling has to be used with respect to nonlinearities.

Hypothesis:

The hypothesis of the MISO method for identification of nonlinear systems is that the nonlinear system does not include any hysteresis, and that the nonlinear system is separable into the linear and nonlinear parts, where the nonlinear part can be also separated to the nonlinear part with memory and linear part that includes only the memory effect of nonlinear system.

2. Nonlinear Systems

A system is called linear, if it is additive and satisfies the homogeneity property (additivity and homogeneity properties are called superposition). If at least one of these principles is not satisfied, the system is called *nonlinear*. It is recalled here what these principles mean:

- *Additivity:* The response to the input signals x_1 and x_2 is equal to the sum of responses to each of the input signals x_1 and x_2 separately
$$\chi[x_1 + x_2] = \chi[x_1] + \chi[x_2].$$
- *Homogenity:* As we increase the input signal x, the output signal $\chi[x]$ is also increased by the same scale factor,
$$\chi[\alpha x] = \alpha\chi[x].$$

In other words, a nonlinear system is one for which principles of superposition (homogeneity and/or additivity) do not hold.

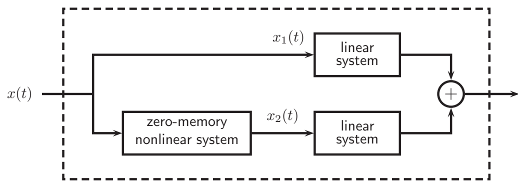


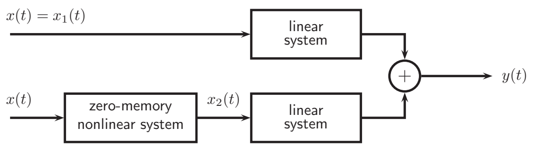
Fig. 1. Nonlinear system with purely linear and zero-memory nonlinear parts.

3. Two Input Single Output Nonlinear System

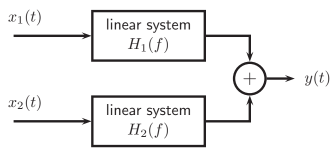
The system described in Figure 1 may be seen as a two-input single-output system with linear Frequency Response Functions and inputs $x_1(t)$, $x_2(t)$, which are such that

$$x_2(t) = g[x_1(t)]. \tag{1}$$

The system can be redrawn as can be seen in Figure 2b and then Figure 2b, which is completely two-input single-output linear system. It is important to note that only the input signals are in nonlinear relation and as a consequence they could be mutually correlated.



(a) with nonlinear part



(b) without nonlinear part

Fig. 2. Two-input single-output system.

The special case of a Multiple Input / Single Outputsystem will now be considered. The number of inputs is reduced to two for better understanding the relations in a MISO system. The input $x_1(t)$ in the Figure 2b is a white Gaussian noise (WGN) which means firstly that its density function and a variance σ_x^2 is normal, with a zero mean value.

$$p(x) = \frac{1}{\sigma_x\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma_x^2}\right), \tag{2}$$

secondly that the stochastic process $x_1(t)$ is stationary and ergodic and lastly that its power spectral density (PSD) is a constant

$$S_{xx}(f) = \sigma_x^2. \tag{3}$$

The input $x_2(t)$ in Figure 2b is a record of a stationary and ergodic stochastic process. These two records may be correlated. Since the signals $x_1(t)$, $x_2(t)$ and $y(t)$ are known, the *frequency response function (FRF)* $H_1(f)$ and $H_2(f)$ may be estimated.
For the two-input single-output system the *cross-spectral density functions* are defined by

$$\begin{aligned} S_{1y}(f) &= H_1(f)S_{11}(f) + H_2(f)S_{12}(f), \\ S_{2y}(f) &= H_1(f)S_{21}(f) + H_2(f)S_{22}(f). \end{aligned} \tag{4}$$

and the transfer functions $H_1(f)$ and $H_2(f)$ are consequently such that [1]

$$\begin{aligned} H_1(f) &= \frac{S_{22}(f)S_{1y}(f) - S_{12}(f)S_{2y}(f)}{S_{11}(f)S_{22}(f) - |S_{12}(f)|^2}, \\ H_2(f) &= \frac{S_{11}(f)S_{2y}(f) - S_{21}(f)S_{1y}(f)}{S_{11}(f)S_{22}(f) - |S_{12}(f)|^2}. \end{aligned} \tag{5}$$

In the special case, $S_{12}(f) = 0$, the inputs are totally uncorrelated, and Equation (5) reduce to

$$\begin{aligned} H_1(f) &= \frac{S_{1y}(f)}{S_{11}(f)}, \\ H_2(f) &= \frac{S_{2y}(f)}{S_{22}(f)}, \end{aligned} \tag{6}$$

which is the representation of a usual single-input single-output system.

4. MISO for Nonlinear Systems using polynomial serie

The nonlinear model described in Figure 1 can be further generalized by increasing the number of nonlinear branches. Each branch consists of a zero-memory nonlinear system, which is followed by a linear system (filter).
The general nonlinear branch of Figure 1 can be expressed as a sum of another nonlinear functions. Note that two different situations may occur. The first corresponds to the estimation of the branches when sufficient a priori based knowledge allows to estimate the input signals to be analyzed (for example input signal $x_n(t) = t^\beta$, for $t > 0$ with $\beta \in [1.6 - 1.8]$). The second one corresponds to a blind identification of the branches where nothing is known. The special case of such a nonlinear system can be the system in Figure 3, where the general nonlinear function is expressed in terms of *Taylor series*. The input signals are then $x_1(t) = x(t)$, $x_2(t) = x^2(t)$, $x_3(t) = x^3(t), \ldots$
From the knowledge of $x_1(t), x_2(t), \ldots, x_q(t)$ the Multiple Input / Single Outputsystem is defined and the linear filters $H_1(f), H_2(f), \ldots, H_q(f)$ can be estimated. The input signal $x_1(t) = x(t)$ is WGN. The other inputs $x_2(t), x_3(t), \ldots$ are signals, which are in zero-memory nonlinear relations with signal $x_1(t)$. Input signals may be correlated and the *cross-spectral density functions* for all the combinations of inputs have to be taken into consideration. The *cross-spectral density functions* between input signal $x_1(t)$ and output signal $y(t)$ is

$$\begin{aligned} S_{1y}(f) &= H_1(f)S_{11}(f) + \\ &+ H_2(f)S_{12}(f) + \\ &\vdots \\ &+ H_q(f)S_{1q}(f), \end{aligned} \tag{7}$$

[1] by supposing that coherence function $\gamma_{12}^2 = \frac{|S_{12}(f)|^2}{S_{11}(f)S_{22}(f)} \neq 1$

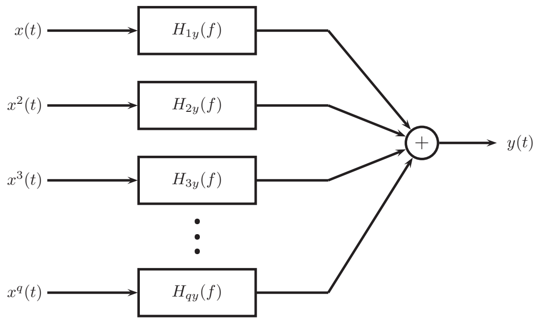


Fig. 3. MISO System for Taylor series.

hence

$$S_{iy}(f) = \sum_{j=1}^{q} H_j(f) S_{ij}(f), \tag{8}$$

for $i = 1, 2, \ldots, q$. This set of equations for a different i can be rewritten into a matrix form for each frequency f, where i is the index of row. Let denote the column matrices $\mathbf{H^f}$ and $\mathbf{S_y^f}$ and the square matrix $\mathbf{S^f}$ as

$$\mathbf{H^f} = \begin{pmatrix} H_1(f) \\ H_2(f) \\ \vdots \\ H_q(f) \end{pmatrix}, \tag{9}$$

$$\mathbf{S_y^f} = \begin{pmatrix} S_{1y}(f) \\ S_{2y}(f) \\ \vdots \\ S_{qy}(f) \end{pmatrix}, \tag{10}$$

$$\mathbf{S^f} = \begin{pmatrix} S_{11}(f) & S_{12}(f) & \cdots & S_{1q}(f) \\ S_{21}(f) & S_{22}(f) & \cdots & S_{2q}(f) \\ & \vdots & & \\ S_{q1}(f) & S_{q2}(f) & \cdots & S_{qq}(f) \end{pmatrix}. \tag{11}$$

Then, the relationships (8) between *auto/cross-spectral density functions* and FRF from Equation (8) can be rewritten as

$$\mathbf{S_y^f} = \mathbf{S^f H^f}. \tag{12}$$

The index $\mathbf{f}$ expressed the frequency, for which the relationships hold.
Now the FRF $H_i(f)$ can be obtained by using the matrix form for all frequencies from

$$\mathbf{H^f} = \mathbf{inv}(\mathbf{S^f})\mathbf{S_y^f}. \tag{13}$$

If a blind identification is supposed, the input signals of a multiple input linear system can be chosen according to polynomial series. Any arbitrary real-valued function $f(x)$ can be expanded in terms of Taylor series as

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n = \sum_{n=0}^{\infty} k_n (x-a)^n, \tag{14}$$

where $a \in \Re$. When the expansion of $f(x)$ is near $a = 0$, the the Taylor series become a Maclaurin series

$$f(x) = \sum_{n=0}^{\infty} c_n x^n. \tag{15}$$

and the approximation we use is using a finite series. In time domain the output signal from such a multiple input system with polynomial series is

$$y(t) \equiv y_q(t) = \sum_{n=0}^{q} c_n x^n(t). \tag{16}$$

The Multiple Input / Single Outputtechnique tends to estimate the linear systems $H_n(f)$, which represent frequency dependent coefficients of Taylor Series. The output signal $y(t)$ of such a system can be expressed as a sum of convolution in time domain

$$y(t) = \sum_{n=0}^{q} h_n(t) * x^n(t), \tag{17}$$

or as a sum of products in frequency domain in terms of Fourier Transform,

$$Y(f) = \sum_{n=0}^{q} H_n(f) \cdot FT\{x^n(t)\}. \tag{18}$$

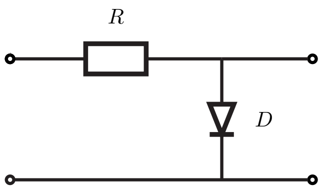
For studying nonlinear systems presenting an ”odd behavior”, it may be useful to consider the set of function $|x|x, |x|x^3, \cdots, |x|x^{2n-1}$ instead of even functions $x^2, x^4, \ldots, x^{2n}$. The Odd series expansion can then be expressed as

$$f(x) = \begin{cases} \sum_{n=1}^{\infty} c_n x^n, & for\ n\ odd, \\ \sum_{n=2}^{\infty} c_n |x| x^{n-1}, & for\ n\ even. \end{cases} \tag{19}$$

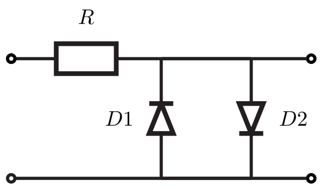
There is a correlation between all inputs of a multiple input system due to the nonlinear relationship between both inputs. The inputs are partially correlated so that the signal of a second branch is partly a linear filtering of the signal from the first (linear) branch. Using the input signal $x(t)$ as a record from a random Gaussian process, and using the polynomial series, the process of decorrelation of input signal is effortless and the input signals can be calculated, so the filtering to predict linear dependency between inputs is no longer need. The process of decorrelation of input signal is beyond the bounds of this article.

5. Identification of a Real Nonlinear System

Two simple nonlinear systems has been tested on MISO method identification to compare the practicality of this method. The first nonlinear system has been a simple circuit with a resistor and a diode (nonlinear device) that is shown in Figure 4a. A nonlinear characteristic of a diode is well known and one can call it a hard nonlinearity. The second nonlinear system has been chosen an electrical circuit with two diodes in parallel - to obtain an odd nonlinear system (Figure 4b). There is also a hard nonlinearity, but this kind of circuit is also known as a limiter, where the circuit is linear till some excitation, and after passing some value, the circuit becomes a nonlinear one.



(a) nonlinear circuit A

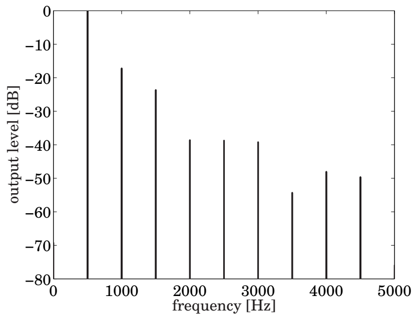


(b) nonlinear circuit B

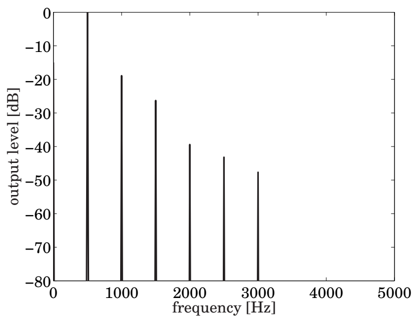
Fig. 4. Nonlinear testing circuit

6. Conclusion

To verify the model and MISO method a comparison between the real circuit and the model has to be realized. The input harmonic signal (frequency 500 Hz) has been put into the real system and into the model and the output spectrum of both has been compared. Results are shown in Figures 5 and 6. The degree of polynomial series was six for both experiments, so in the case of classic Taylor series only first six harmonics can be seen. In the case of the odd nonlinear system the harmonics of even order are not included into the model (because of odd nonlinear system). In the real measurement there are harmonic components of even order caused by the different diodes, but for modeling there were neglected.

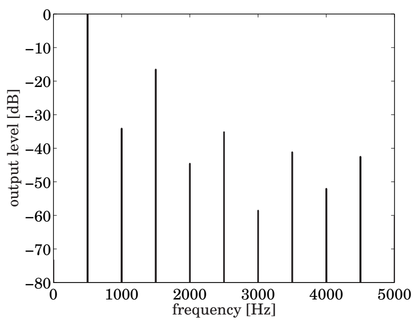


(a) real circuit

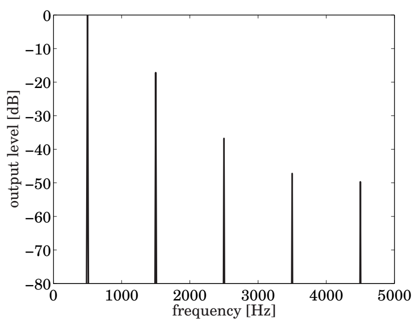


(b) model

Fig. 5. Comparison of model and experiment - output spectrum of nonlinear system A



(a) real circuit



(b) model

Fig. 6. Comparison of model and experiment - output spectrum of nonlinear system B

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Antonín Novák, born on 15th July, 1981, is a post-graduate at the Czech Technical University in Prague, Faculty of Electrical Engineering and at Université du Maine, Le Mans as part of a PhD. Cotutelle programme. His research interests include analysis and measurement of non-linear systems and digital signal processing. He received an Ing. degree in engineering from the Czech Technical University in 2006 at the departement of Radioelectronics and, in the same year, an equivalence of Master 2 Recherche from Ecole Doctorale de l’Univeristé du Maine.