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IDENTIFICATION OF NONLINEARITY OF ELECTRO-ACOUSTIC SYSTEMS USING A DIRECT PATH MISO METHOD

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ABSTRACT
The work deals with the Direct Path Multiple-Input Single-Output (MISO) method adapted for the identification of Nonlinearity of Electro-acoustic Systems. The method is based on a blind identification which uses decorrelated power series expansion, without having any knowledge of the shape of the nonlinear function. The nonlinear Direct Path model is represented by an equivalent Multiple-Input Single-Output linear model, where the inputs are nonlinear contributions of the original input signal. Each branch of a complex nonlinear model, with its nonlinear input, represents a "separable nonlinearity" or a static nonlinearity that is followed by a linear system so that the memory effect is represented by a linear filter. The input signal for identification is a record from stationary Gaussian random process. The method has been numerically tested on static nonlinear systems, such as limiter and death-zone systems. Also, an experiment on real electro-acoustic system has been performed.

INTRODUCTION
Almost all the systems we can find in the domain of acoustics and electro-acoustics behave more or less nonlinearly. There is a different behavior at low and high amplitudes of input signals that is accompanied by a presence of additional spectral components in output signal which are not in the input signal. The nonlinearity in such an audio system may cause either a distortion of the sound which has a disagreeable effect on its perception, or a change in timbre of the sound that is commonly used in music. From that point of view, we can divide nonlinear electro-acoustic systems into two categories. First, where the nonlinear behavior is not desirable and the system is supposed to be linear – e.g. a loudspeaker, an amplifier, etc. The second category consists of nonlinear systems to which the nonlinearity is introduced on purpose to change the timbre of the sound - such as an audio limiter for an electric guitar.

Behavior of nonlinear systems cannot be generalized, thus a single nonlinear modeling method cannot be applied for all the nonlinear systems. One of the easiest ways to detect nonlinearities is to excite the system with pure sinusoids and describe the nonlinearity by a coefficient of nonlinear harmonic distortion. But there exist nonlinear systems, where the harmonic distortion measurement is not useful: for example the Doppler Effect, where the nonlinear distortion is caused by a change in wavelength as a result of motion between the diaphragm of the loudspeaker and the receiver. A single sinusoid tone cannot produce higher-order harmonics, so an inter-modulation measurement has to be performed [1]. The goal of this work is to identify and to model the nonlinearities of electro-acoustic systems for better comprehension of their principles. Once the model is set up, the nonlinear distortions can be possibly eliminated.

In the following paper, the static and dynamic nonlinear systems are defined and the basics of Direct Path Multiple Input Single Output using power series are described. To show the efficiency of the MISO method, two static nonlinear systems are identified and finally a real identification of electro-acoustic system is performed.

NONLINEAR SYSTEM

Generally speaking, a linear system is such that principles of superposition (homogeneity and additivity) are satisfied. If at least one of these conditions does not hold the system is nonlinear.
In general, nonlinear systems can be classified into two types – static nonlinearity and nonlinearity with memory. A static nonlinear system is a nonlinear system where the output signal $y(t)$ is an instantaneous (non-linear) function of the input signal $x(t)$, noted $g[x(t)]$.

$$y(t) = g[x(t)] \quad \text{(Eq. 1)}$$

Static nonlinear system is also called "zero-memory" because such a system does not weight past inputs (has not any memory) and the current value of output signal is a function of current value of the input signal.
For the MISO identification method a finite memory system is used. Finite memory system weights past inputs (has a memory) and the current value of output signal is a function of both current and past values of the input. Special physical case of a finite memory nonlinear system which is partly used in the MISO method is where the memory is expressed by a linear filter preceded by a zero-memory nonlinear system, see Figure 1.

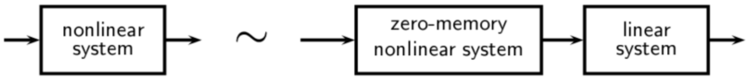


Figure 1.- Separation of memory of finite memory nonlinear system

DIRECT PATH MULTIPLE INPUT SINGLE OUTPUT SYSTEM - POWER SERIES MODEL

Employing the Multiple Input Single Output System paradigm, the zero-memory nonlinear system followed by a linear system can be expanded into a parallel set of such systems. This way, there is a different memory effect expressed by a linear system $H_n(f)$ for each nonlinear system, as shown in Figure 2. In other words, each harmonic component may have a different frequency dependent attenuation; this behavior is expectable in electro-acoustic systems.

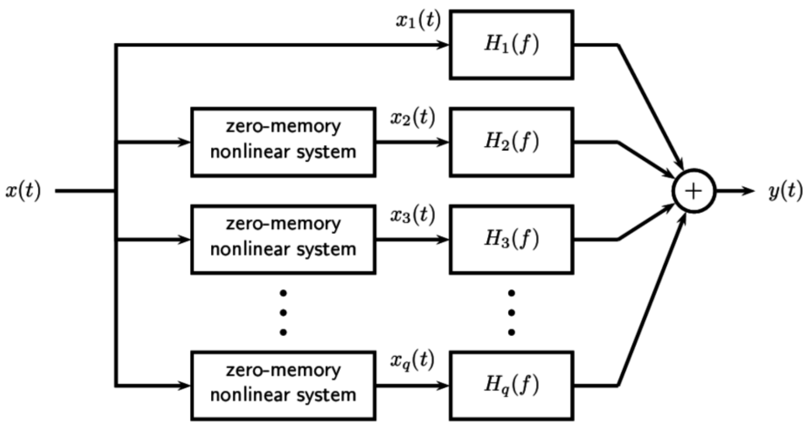


Figure 2.- Multiple Input Single Output nonlinear model

In this paper only the power series model is mentioned, nevertheless the MISO method can use any number of nonlinear inputs with any shape of static nonlinear function. The blind identification can be performed to obtain preliminary results of the power series model; this information can be subsequently utilized to investigate the nonlinearity using a more suitable set of nonlinear functions.

Identification and Decorrelation of Multiple Inputs

For identification of nonlinear systems using MISO nonlinear model we use a white noise excitation, where the input signal is a record from stationary Gaussian random process. That means firstly that its density function is normal, with zero mean value and variance σ_x^2. Furthermore, the stochastic process is stationary and ergodic and lastly its power spectral density is constant. Subsequently the estimation of the linear filters from MISO model (see Figure 2) using power and cross spectral densities is performed as all the nonlinear inputs are known [2].

Input signals $x_n(t)$ from the MISO model may be mutually correlated which results in cross-spectral density functions. Let suppose that the two inputs of Two Inputs Single Output System (Figure 3) are partially correlated so that the signal $x_2(t)$ is partly linearly filtered signal $x_1(t)$. This filtering may be modeled by a linear filter $L_{12}(f)$ placed between inputs as shown in Figure 3.

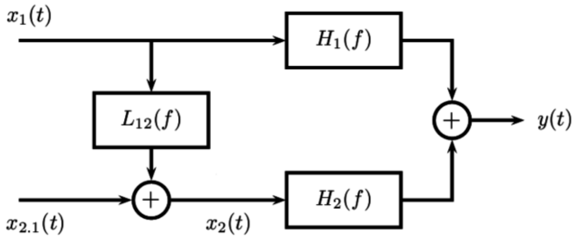


Figure 3.- Two-input, single-output system with correlated inputs

As the input signal $x(t)$ is a white noise and the power series are used, the linear dependency between the inputs is expressed by a scale factor (*k* = constant). Thus the decorrelated input signal $x_{2.1}(t)$, which has no correlation with the input signal $x_1(t)$, can be obtained.

For example of the Two Inputs Single Output System with inputs $x_1(t)$ and $x_3(t)$ (indexes are chosen according to power series)

$$x_1(t) = x(t)$$
$$x_3(t) = x^3(t)$$

the decorrelate signal $x_{3.1}(t)$ can be expressed as

$$x_{3.1}(t) = x^3(t) - 3\sigma_x^2 x(t). \quad \text{(Eq. 2)}$$

The scale factor $3\sigma_x^2$ is obtained using Bussgang theorem [3]. This theorem tells us that for a Gaussian input signal with known power-spectral density function passing through any zero-memory nonlinear system, the input/output cross-spectral density function is directly proportional to the power-spectral density function. If the power-spectral and cross-spectral density are directly proportional with no frequency dependence, the linear filter L_{12} between the two input signals reduces to a constant. The following equation shows how the coefficient of decorrelation is obtained using the Bussgang theorem:

$$L_{13} = \frac{S_{x^3x}}{S_{xx}} = \frac{(E[xx^3]/\sigma_x^2)S_{xx}}{S_{xx}} = \frac{E[xx^3]}{\sigma_x^2} = \frac{E[x^4]}{\sigma_x^2} = \frac{3\sigma_x^4}{\sigma_x^2} = 3\sigma_x^2, \quad \text{(Eq. 3)}$$

where $E[\ \ ]$ denotes an ensemble average over the set of records.

If another input is added, the process of decorrelation has to be performed between all the inputs. To obtain completely decorrelated set of input signals of an arbitrary MISO model, the decorrelation between all the input signals of power series is necessary. The procedure to decorrelate all the inputs is again based on the Bussgang theorem.

STATIC NONLINER SYSTEM IDENTIFICATION USING THE MISO METHOD

When using the MISO method for identification of a static nonlinear system, the linear filters $H_1(f)$, $H_2(f)$, … , $H_n(f)$ used in the MISO nonlinear model are not frequency dependent and their constant values are given by the coefficients of power series. As an example of identification of such a system, two well known static nonlinear systems have been chosen: Limiter and Death-Zone system.

Limiter

A limiter is a nonlinear system with a given value A of saturation, below which the input signal is directly passed to the output. Input signal values exceeding the saturation are cut off, that results in a distortion of the signal. The input-output (I/O) characteristics of a limiter is shown in Figure 4; the theoretical I/O function is plotted in dashed line whilst the solid curve represents the reconstruction of I/O characteristics after identification using 6 order power series.

$$y = \begin{cases} -A, & x \leq -A \\ x, & |x| \leq A \\ A, & x \geq A \end{cases} \tag{Eq. 4}$$

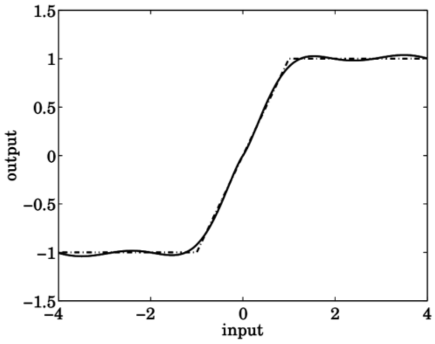


Figure 4. Input Output Characteristics of Limiter

Death-Zone system

Death-Zone system can be conceptualized as an opposite case of the Limiter. In the case of the Limiter – the higher the amplitude of the input signal (after passing a saturation value), the higher the distortion of the output signal. In the case of a Death-Zone system – the higher the amplitude of input signal, the lower the distortion. The shape of the input-output characteristics is shown in Figure 5. As in Figure 4, the theoretical I/O curve and its reconstruction are shown.

$$y = \begin{cases} b(x+A), & x \leq -A \\ 0, & |x| \leq A \\ b(x-A), & x \geq A \end{cases} \tag{Eq. 5}$$

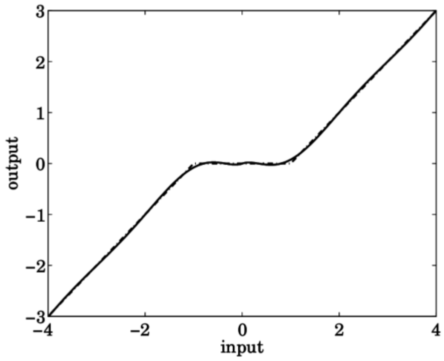


Figure 5.- Input Output Characteristics of Dead-zone system

MEASURING OF A REAL AUDIO SYSTEM

The above described method has been tested on an Audio Limiter. The test signal was a white noise with standard deviation 0.2 (maximum value not exceeding 1) and zero mean value. The sampling frequency was $f_s = 192\ \mathrm{kHz}$. The resulting model characteristics are shown in Figure 6; the magnitudes of individual linear filters of the MISO model are plotted. These filters are those of the decorrelated model, that means the zero-memory systems from the model (see Figure 1.) were already decorrelated. The correct functionality of the model can be verified by eye: the reconstruction of the response to a sinusoidal signal and the I/O characteristics obtained from the estimated model are depicted in Figure 7.

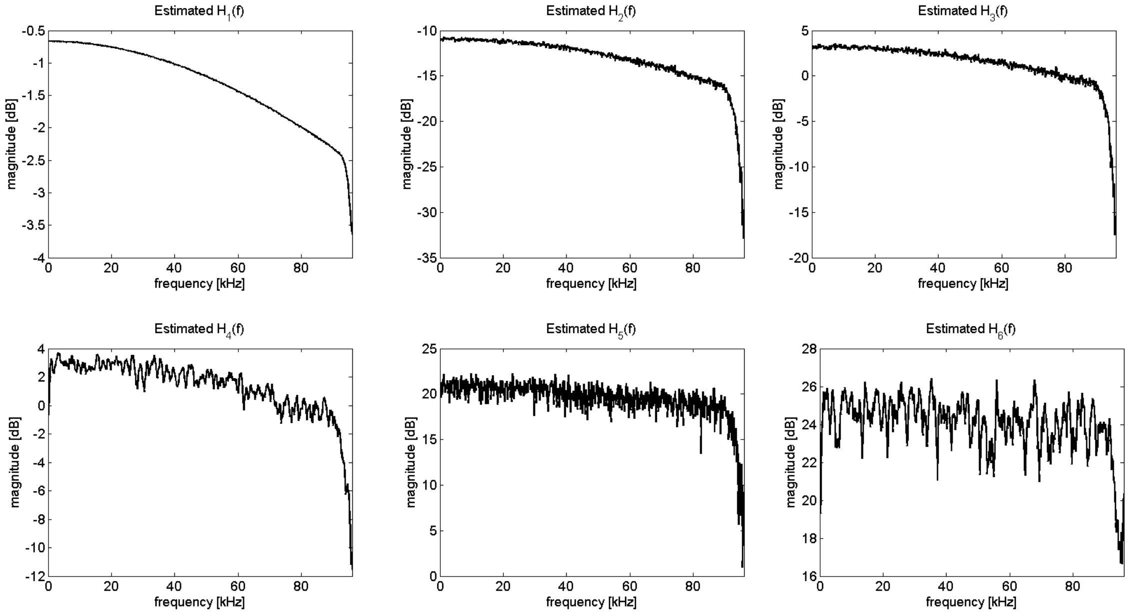
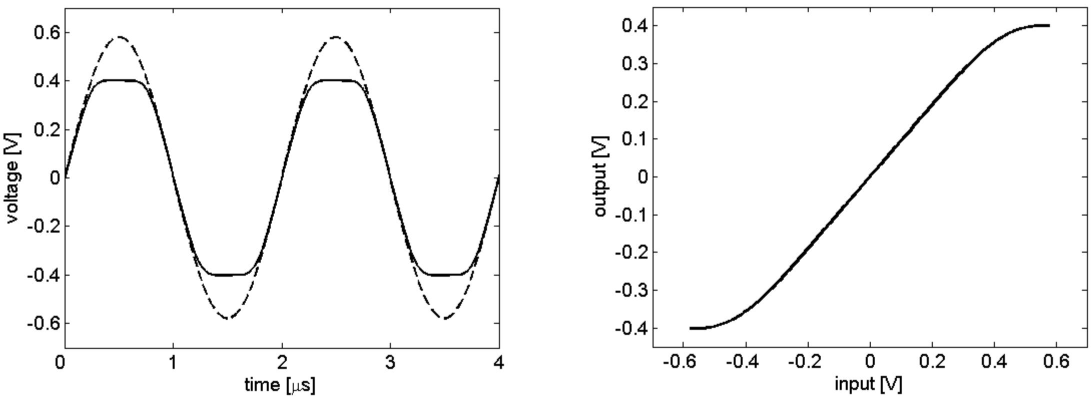


Figure 6.- Linear filters $H_1(f)$, $H_2(f)$, … , $H_n(f)$, of MISO model of the Audio Limiter



a) b)

Figure 7.- Reconstruction, using a sine signal; a) input sine signal (dashed) and output distorted signal (solid) of MISO model of the Audio Limiter; b) input-output characteristics of MISO model of the Audio Limiter

CONCLUSIONS

The aim of this study is to show that the Direct Path Multiple Input Single Output method can be advantageously used for nonlinear system analysis. The method based on a blind identification without having any knowledge of the shape of the nonlinear function has been tested on two exemplary cases of nonlinear systems – Limiter and Death-Zone system. It was shown that in the simple case of static nonlinearities the method can identify even a hard nonlinearity. Nevertheless, the MISO method can successfully identify also the dynamic nonlinear systems with memory expressed by a linear system. A real Audio Limiter (nonlinear system) was analyzed using the MISO method. After obtaining the MISO model parameters, the model response to a harmonic signal was acquired to verify the accuracy of the identification. The MISO model predicts well the nonlinear behavior of the Audio Limiter as seen from the I/O characteristics in Figure 7b. The main advantage of this method is the usage of white noise as the input signal, so the measurement is performed at all frequencies (supposing the span is comparable to the measured frequency band) and at all amplitude levels (limited by the variation of the input signal). On the other hand, the usage of the white noise requires longer averaging time to achieve the desired accuracy and to avoid bias error.

ACKNOWLEDGEMENTS

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